MAU34401 Homework Problem Sheet 2
Submit via Blackboard before 5pm, 2nd November, 2020. [1]

Policy

Extensions of up to a week will be granted upon request before the due deadline and for sufficient cause.
Please include an estimate of the time taken to complete the set and remember to put your name on the solutions.

Please read the following declaration:

I have read and I understand the plagiarism provisions in the General Regulation s of the University Calendar for the current year, found at http://www.tcd.ie/ca lendar.
I have also completed the Online Tutorial on avoiding plagiarism "Ready Steady Write", located at http://tcd-ie.libguides.com/plagiarism/ready-steady-write.

Then sign this sheet and include it with your solutions.

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Signature Date

1. Consider a conducting sphere, radius a, centred at the origin. A positive charge $+q$ is located at $z = -R$ and a charge $-q$ is located at $z = R$.

   (a) Find a Green's function for the problem and use it to determine the potential outside the sphere.

   (b) Show that the charge density on the sphere is $\sigma = 3\epsilon_0 E_0 \cos\theta$ where $E_0 = 2q/(4\pi\epsilon_0 R^2)$.

[1] Sinéad Ryan, see http://www.maths.tcd.ie/˜ryan/34401.html

2. Consider a spherical shell of radius a with a missing cap at the north pole - defined by the cone with opening angle α and with a uniform charge distribution, σ. The charge distribution in spherical coordinates is

$$\rho(r, \theta, \phi) = \sigma\delta(r - a)\Theta(\cos\alpha - \cos\theta),$$

where $\Theta(x)$ is the Heaviside function [defined by $\Theta(x) = 0$ for $x < 0$ and $\Theta(x) = 1$ for $x > 0$].

Sketch the sphere. Find the potential inside and outside of the spherical surface.

3. The associated Legendre equation (see lecture notes), is a second order differential equation and as such has 2 linearly independent solutions: the *associated Legendre functions of the first and second kind.*

Denoting these as $P_l^m(x)$ and $\widetilde{P}_l^m(x)$ note that for non-zero m one solution, say $\widetilde{P}_l^m(x)$ diverges as $x \to \pm 1$ and for general m the Rodrigues' formula gives the $P_l^m(x)$ (for $m \geq 0$):

$$P_l^m(x) = \frac{(-1)^m}{2^l l!}(1 - x^2)^{m/2}\frac{d^{l+m}}{dx^{l+m}}(x^2 - 1)^l.$$

- Comment on the allowed values for m and l so that solutions to the associated Legendre equation remain finite.
- Write down the first few polynomials, specfically $P_0^0(x), P_1^0(x), P_1^1(x), P_2^1(x), P_2^2(x), P_3^1(x), P_3^2(x)$ and use mathematica (or your favourite plotting package) to plot these.

  A screenshot or any other method is fine for submission of this part.
- The Rodriques' formula is also valid for negative m if $|m| \leq l$. Show that $P_l^{-m}(x) = (-1)^m \frac{(l-m)!}{(l+m)!} P_l^m(x)$.

  Hint: you could apply Leibnitz's formula to $(x^2 - 1)^l$.