UNIVERSITY OF DUBLIN

XMA1M01

TRINITY COLLEGE

Faculty of Science

school of mathematics

JF Natural Sciences **Trinity Term 2010**

Module 1M01

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Credit will be given for the best THREE questions attempted IN EACH SECTION.
Use a different answer book for each section.

Log tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination.
Please indicate the make and model of your calculator on each answer book used.

SECTION A

1. Consider the matrix

$$A = \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}.$$

   (a) [14 marks] Use Gauss-Jordan elimination to find the inverse of the matrix A.

   (b) [6 marks] Verify that your result is correct

2. Consider the following statement:

   Theorem: Suppose the largest eigenvalue of a Leslie matrix is r. Then the relative growth rate of the population described approaches r as the number of years approaches infinity ie the long-term growth rate is the largest eigenvalue of the system.

   Now, given the Leslie matrix for an ovenbird population in central Missouri is

$$G = \begin{pmatrix} 0.728 & 1.302 \\ 0.52 & 0.62 \end{pmatrix} \tag{1}$$

   (a) [6 marks] What is the defining equation of an eigenvalue λ of an $n \times n$ matrix A.

   (b) [14 marks] Find the long-term (percentage) growth rate from the Leslie matrix G given here.

3. Consider the following experiment. A box contains five tickets labelled

$$\boxed{1}, \quad \boxed{0}, \quad \boxed{0}, \quad \boxed{0}, \quad \boxed{1}$$

   Six tickets are drawn randomly from this box with replacement. That means that a ticket is drawn, observed and then replaced before the next ticket is drawn.

   (a) [5 marks] Define a binomial experiment.

   (b) [5 marks] Explain why the experiment described is binomial.

   (c) [5 marks] Find the probability that the sum of the six tickets drawn is 3.

   (d) [5 marks] What is the standard deviation for this experiment?

4. Consider a large class of students. Each student in the class is given an identical box containing three tickets, marked $\diamondsuit$, $\heartsuit$ and $\square$. Students are asked to draw two tickets from the box without replacing the first ticket back into the box. A prize is given to those who draw a $\diamondsuit$ first and a $\square$ second.

   (a) [3 marks] State the principle of equally likely outcomes.

(b) [5 marks] State the multiplication rule for the probability that two events, A and B, both occur. Explain what the probability $P(B|A)$ means.

(c) [4 marks] Considering the experiment described above, what proportion of students draw the $\diamondsuit$ ticket first?

(d) [4 marks] Of the students that drew the $\diamondsuit$ first, what proportion draw the $\square$ second?

(e) [4 marks] What proportion of the students in the class will win the prize?

SECTION B

5. (a) [6 marks] Find an equation for the straight line which is parallel to $y = 3x - 4$ and passes through the point $(1, 2)$.

   (b) [7 marks] Simplify the expression $\sqrt[4]{16x^3e^{\ln(x)}}$.

   (c) [7 marks] What is the average rate of change of the function $f(x) = x^2 - 3x$ as x varies from -1 to 2?

6. (a) Find the derivatives of the following expressions:

   (i). [4 marks] $\dfrac{\sin(x)}{1+x}$

   (ii). [4 marks] $5 - 6e^{1+x^2} + x\ln(x)$

   (iii). [4 marks] $(x^2 + \sin(2\pi x))^{0.3}$

   (b) [8 marks] Evaluate the indefinite integral $\displaystyle\int e^x - 4x\sin(x^2)\,dx$.

7. Let $f(x) = 12x - x^3 + 10$.

   (a) [10 marks] Compute the coordinates of the absolute maximum and the absolute minimum of $f(x)$ over the interval $[0, 3]$.

   (b) [10 marks] What is the average value of $f(x)$ over the interval $[0, 3]$?

8. The number of bacteria $P(t)$ in a petri dish grows according to

$$P(t) = (3 \times 10^5)e^{0.04t}$$

where t is the time, in hours, since the bacteria were introduced to the petri dish.

   (a) [2 marks] How many bacteria were introduced to the petri dish?

   (b) [4 marks] What is the generation time for this population?

   (c) [7 marks] Compute the instantaneous rate of change of $P(t)$ 24 hours after the bacteria were introduced to the petri dish.

   (d) [7 marks] When does the population reach 2×10^6 bacteria?