

Mathematics Tutorial Sheet ¹

1. Consider the 2×2 Leslie matrix

$$A = \begin{pmatrix} 0 & 2 \\ 0.25 & 0.5 \end{pmatrix}.$$

- (a) Given a population vector $P = \begin{pmatrix} 100 \\ 160 \end{pmatrix}$ in year N , use matrix-vector multiplication to determine the population in year $N + 1$.
- (b) Determine the eigenvalues of A .
- (c) Recall that the long-term growth rate of a population is given by the largest eigenvalue of the corresponding Leslie matrix and the stable age population is given by its corresponding eigenvector. Determine the stable-age population for the Leslie matrix, A , given above.

2. Determine the eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 6 & 16 \\ -1 & -4 \end{pmatrix}.$$

Starting from $AP = PD$ where P is the matrix whose columns are the eigenvectors and D is the diagonal matrix of eigenvalues, show that $A^k = PD^kP^{-1}$.

Find A^4 .

3. Use the Cayley Hamilton theorem to determine A^5 given,

$$A = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix} \tag{1}$$

Write down an expression for A^4 .

¹Sinéad Ryan, ryan@maths.tcd.ie, <http://www.maths.tcd.ie/~ryan/MA1M01.html>