

Q1

$$B = \begin{pmatrix} 3 & -1 & 7 \\ 10 & 1 & -8 \\ -5 & 2 & 4 \end{pmatrix}$$

$$\begin{aligned} \det(B) &= 3 \begin{vmatrix} 1 & -8 \\ 2 & 4 \end{vmatrix} - (-1) \begin{vmatrix} 10 & -8 \\ -5 & 4 \end{vmatrix} + 7 \begin{vmatrix} 10 & 1 \\ -5 & 2 \end{vmatrix} \\ &= 3 [1(4) - (-8)(2)] + 1 [10(4) - (-8)(-5)] + 7 [10(2) - 1(-5)] \\ &= \underline{235} \end{aligned}$$

$$\det(C) = 1(-8) - (-12)(10) = -8 + 120 = \underline{112}$$

$$\det(F) = 0(-1) - 4(0) = \underline{0}$$

$$\begin{aligned} \det(A) &= -1 \begin{vmatrix} 2 & -1 \\ 4 & 7 \end{vmatrix} - 4 \begin{vmatrix} 6 & -1 \\ 7 & 7 \end{vmatrix} + 9 \begin{vmatrix} 6 & 2 \\ 7 & 4 \end{vmatrix} \\ &= -1 [14 - 4] - 4 [42 - 7] + 9 [24 - 14] \\ &= \underline{-124} \end{aligned}$$

Q2

$$A = \begin{pmatrix} 3 & 4 \\ 7 & 10 \end{pmatrix}$$

CRAMERS RULE: $\det(A) = 3(10) - 4(7) = 2$

$$A^{-1} = \frac{1}{2} \begin{pmatrix} 10 & -4 \\ -7 & 3 \end{pmatrix}$$

GJE

$$\left(\begin{array}{cc|cc} 3 & 4 & 1 & 0 \\ 7 & 10 & 0 & 1 \end{array} \right) \xrightarrow{R_1/3} \left(\begin{array}{cc|cc} 1 & 4/3 & 1/3 & 0 \\ 7 & 10 & 0 & 1 \end{array} \right)$$

$$\xrightarrow{R_2 - 7R_1} \left(\begin{array}{cc|cc} 1 & 4/3 & 1/3 & 0 \\ 0 & 2/3 & -7/3 & 1 \end{array} \right) \xrightarrow{\frac{3}{2} \cdot R_2} \left(\begin{array}{cc|cc} 1 & 4/3 & 1/3 & 0 \\ 0 & 1 & -7/2 & 3/2 \end{array} \right)$$

$$\xrightarrow{R_1 - \frac{4}{3}R_2} \left(\begin{array}{cc|cc} 1 & 0 & 5 & -2 \\ 0 & 1 & -7/2 & 3/2 \end{array} \right)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 5 & -2 \\ -7/2 & 3/2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 10 & -4 \\ -7 & 3 \end{pmatrix} \text{ à la Cramer.}$$

Q3mx. eqⁿ is

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \\ 7 \end{pmatrix}$$

CJE

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ -6 \\ 7 \end{pmatrix} \xrightarrow{R_2 - R_1} \begin{pmatrix} 1 & 1 & 1 \\ 0 & -3 & 2 \\ 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ -10 \\ 7 \end{pmatrix} \xrightarrow{R_3 - 2R_1} \begin{pmatrix} 1 & 1 & 1 \\ 0 & -3 & 2 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 4 \\ -10 \\ -1 \end{pmatrix}$$

$$\xrightarrow{+3R_3 + R_2} \begin{pmatrix} 1 & 1 & 1 \\ 0 & -3 & 2 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 4 \\ -10 \\ -13 \end{pmatrix} \xrightarrow{R_3 \times (-1)} \begin{pmatrix} 1 & 1 & 1 \\ 0 & -3 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ -10 \\ 13 \end{pmatrix}$$

$$\xrightarrow{R_2 - 2R_3} \begin{pmatrix} 1 & 1 & 1 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ -36 \\ 13 \end{pmatrix} \xrightarrow{-\frac{1}{3}R_2} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 12 \\ 13 \end{pmatrix}$$

$$\xrightarrow{R_1 - R_3} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -9 \\ 12 \\ 13 \end{pmatrix} \xrightarrow{R_1 - R_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -21 \\ 12 \\ 13 \end{pmatrix}$$

$$\Rightarrow x = -21, y = 12, z = 13$$

Check

$$\text{eq}^n 1: -21 + 12 + 13 = -21 + 25 = 4 \checkmark$$

$$\text{eq}^n 2: -21 - 2(12) + 3(13) = -21 - 24 + 39 = -6 \checkmark$$

$$\text{eq}^n 3: 2x + 2y + z = 2(-21) + 2(12) + 13 = -42 + 36 + 13 = 7 \checkmark$$