MAU11002: Mathematics Tutorial Sheet [1]

1. **Determinants** Consider the matrices

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad ; \quad B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

The formulae for the determinants of A and B are

$$\begin{aligned} \det(A) &= a_{11}a_{22} - a_{12}a_{21} \\ \det(B) &= b_{11}\begin{vmatrix} b_{22} & b_{23} \\ b_{32} & b_{33} \end{vmatrix} - b_{12}\begin{vmatrix} b_{21} & b_{23} \\ b_{31} & b_{33} \end{vmatrix} + b_{13}\begin{vmatrix} b_{21} & b_{22} \\ b_{31} & b_{32} \end{vmatrix} \end{aligned}$$

where the straight lines, || indicate a determinant should be calculated.

Use these formulae to calculate the determinants of the following 4 matrices.

$$B = \begin{pmatrix} 3 & -1 & 7 \\ 10 & 1 & -8 \\ -5 & 2 & 4 \end{pmatrix} \quad ; \quad D = \begin{pmatrix} -1 & 4 & 9 \\ 6 & 2 & -1 \\ 7 & 4 & 7 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & -12 \\ 10 & -8 \end{pmatrix} \quad ; \quad F = \begin{pmatrix} 0 & 4 \\ 0 & -1 \end{pmatrix}$$

2. **Cramer's rule, determinants and inverses**

Cramer's rule is a useful way to calculate 2×2 matrix inverses. It states that a matrix

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad \text{has inverse} \quad A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}.$$

Show that you get the same answer using Cramer's Rule and Gauss Jordan elimination when calculing the inverse of

$$A = \begin{pmatrix} 3 & 4 \\ 7 & 10 \end{pmatrix}.$$

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3. **Systems of equations and matrix inverse**

Given the following system of linear equations, write the corresponding matrix equation and use Gauss Jordan elimination to solve for x, y, z

$$\begin{aligned} x + y + z &= 4 \\ x - 2y + 3z &= -6 \\ 2x + 3y + z &= 7 \end{aligned}$$