

MAU11002: Mathematics Tutorial Sheet 2 ¹

1. Warm-up matrix multiplication exercise

Determine the following matrix products: AB, BC given

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 4 & 3 & 7 \end{pmatrix}; B = \begin{pmatrix} 1 & 0 & 1 & 2 \\ 3 & 2 & 5 & 5 \\ 6 & 4 & 3 & 2 \end{pmatrix}; C = \begin{pmatrix} 2 \\ 6 \\ 9 \\ 1 \end{pmatrix}.$$

Write down in advance the size of the product matrices, AB and BC .

2. Leslie matrices

Given a Leslie matrix, G , and a population vector v written

$$G = \begin{pmatrix} 1.2 & 2.5 \\ 0.8 & 0.8 \end{pmatrix} \quad \text{and} \quad v = \begin{pmatrix} 240 \\ 124 \end{pmatrix}, \quad (1)$$

Determine the population in years 2 and 3. Use matrix-vector multiplication!

3. Matrix properties

The *identity matrix* is a square matrix with 1's on the diagonal and zero elsewhere, eg the 2×2 identity matrix is

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Show that $IA = AI = A$ using

$$A = \begin{pmatrix} 6 & 4 \\ 2 & 5 \end{pmatrix}.$$

4. Matrix properties again

Consider the matrices

$$A = \begin{pmatrix} 6 & 4 \\ 2 & 5 \end{pmatrix}; B = \begin{pmatrix} 3 & 5 \\ -4 & 1 \end{pmatrix}.$$

By writing down the transpose of A and B , state if either is symmetric.

Show by direct calculation that $(A^T)^T = A$ and $(AB)^T = B^T A^T$.

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