1MA01: Mathematical Methods Tutorial Sheet 1 [1]

1. **Matrix Addition**

Given three 2×2 matrics,

$$A = \begin{pmatrix} 6 & 4 \\ 2 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 5 \\ -4 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 8 & 3 \\ 0 & -3 \end{pmatrix}. \tag{1}$$

Show by computation, that

(a) $A + (B + C) = (A + B) + C$

Compute

(a) $A - B$

(b) $B - C$

(c) $A + C$

$$\begin{aligned} A + (B + C) &= \begin{pmatrix} 6 & 4 \\ 2 & 5 \end{pmatrix} + \left(\begin{pmatrix} 3 & 5 \\ -4 & 1 \end{pmatrix} + \begin{pmatrix} 8 & 3 \\ 0 & -3 \end{pmatrix} \right) \\ &= \begin{pmatrix} 6 & 4 \\ 2 & 5 \end{pmatrix} + \begin{pmatrix} 11 & 8 \\ -4 & -2 \end{pmatrix} \\ &= \begin{pmatrix} 17 & 12 \\ -2 & 3 \end{pmatrix} \end{aligned}$$

and

$$\begin{aligned} (A + B) + C &= \left(\begin{pmatrix} 6 & 4 \\ 2 & 5 \end{pmatrix} + \begin{pmatrix} 3 & 5 \\ -4 & 1 \end{pmatrix} \right) + \begin{pmatrix} 8 & 3 \\ 0 & -3 \end{pmatrix} \\ &= \begin{pmatrix} 9 & 9 \\ -2 & 6 \end{pmatrix} + \begin{pmatrix} 8 & 3 \\ 0 & -3 \end{pmatrix} \\ &= \begin{pmatrix} 17 & 12 \\ -2 & 3 \end{pmatrix} \end{aligned}$$

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as required.

$$
\begin{aligned}
A - B &= \begin{pmatrix} 6 & 4 \\ 2 & 5 \end{pmatrix} - \begin{pmatrix} 3 & 5 \\ -4 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ 6 & 4 \end{pmatrix} \\
B - C &= \begin{pmatrix} 3 & 5 \\ -4 & 1 \end{pmatrix} - \begin{pmatrix} 8 & 3 \\ 0 & -3 \end{pmatrix} = \begin{pmatrix} -5 & 2 \\ -4 & 4 \end{pmatrix} \\
A + C &= \begin{pmatrix} 6 & 4 \\ 2 & 5 \end{pmatrix} + \begin{pmatrix} 8 & 3 \\ 0 & -3 \end{pmatrix} = \begin{pmatrix} 14 & 7 \\ 2 & 2 \end{pmatrix}
\end{aligned}
$$

2. **Matrix Transpose**

If A is a matrix, the *transpose* of A is written A^T and it is the matrix obtained by swapping the rows and columns of A. Eg. a matrix,

$$A = \begin{pmatrix} 2 & 3 & 8 \\ 1 & 2 & 5 \end{pmatrix} \tag{2}$$

has a transpose,

$$A^T = \begin{pmatrix} 2 & 1 \\ 3 & 2 \\ 8 & 5 \end{pmatrix} \tag{3}$$

Given a matrix

$$B = \begin{pmatrix} -3 & 2 \\ 2 & 4 \\ -1 & 5 \end{pmatrix} \tag{4}$$

state what size (dimensions) the transpose of B (ie. B^T) will have. Write down B^T.

$$B^T = \begin{pmatrix} -3 & 2 & -1 \\ 2 & 4 & 5 \end{pmatrix}$$

and so B^T is a 2×3 matrix.

3. **Matrix Multiplication**
Given three 2×2 matrics,

$$A = \begin{pmatrix} 6 & 4 \\ 2 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 5 \\ -4 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 8 & 3 \\ 0 & -3 \end{pmatrix}. \tag{5}$$

Show, by computation, that

(a) $AB \neq BA$ (ie. matrix multiplication is *not commutative*)

(b) $A(B+C) = AB + AC$ (ie. matrix multiplication is *distributive* over matrix addition)

(a) First AB

$$\begin{aligned} AB &= \begin{pmatrix} 6 & 4 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} 3 & 5 \\ -4 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 6 \times 3 + 4 \times (-4) & 6 \times 5 + 4 \times 1 \\ 2 \times 3 + 5 \times (-4) & 2 \times 5 + 5 \times 1 \end{pmatrix} \\ &= \begin{pmatrix} 2 & 34 \\ -14 & 15 \end{pmatrix} \end{aligned}$$

Now for BA

$$\begin{aligned} BA &= \begin{pmatrix} 3 & 5 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} 6 & 4 \\ 2 & 5 \end{pmatrix} \\ &= \begin{pmatrix} 3 \times 6 + 5 \times 2 & 3 \times 4 + 5 \times 5 \\ -4 \times 6 + 1 \times 2 & -4 \times 4 + 1 \times 5 \end{pmatrix} \\ &= \begin{pmatrix} 28 & 37 \\ -22 & -11 \end{pmatrix} \end{aligned}$$

Proving that $AB \neq BA$.

(b) Consider the left hand side $A(B+C)$

$$\begin{aligned} B + C &= \begin{pmatrix} 3 & 5 \\ -4 & 1 \end{pmatrix} + \begin{pmatrix} 8 & 3 \\ 0 & -3 \end{pmatrix} \\ &= \begin{pmatrix} 3+8 & 5+3 \\ -4+0 & 1+(-3) \end{pmatrix} = \begin{pmatrix} 11 & 8 \\ -4 & -2 \end{pmatrix} \end{aligned}$$

And then

$$\begin{aligned} A(B+C) &= \begin{pmatrix} 6 & 4 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} 11 & 8 \\ -4 & -2 \end{pmatrix} \\ &= \begin{pmatrix} 6(11)+4(-4) & 6(8)+4(-2) \\ 2(11)+5(-4) & 2(8)+5(-2) \end{pmatrix} = \begin{pmatrix} 50 & 40 \\ 2 & 6 \end{pmatrix} \end{aligned}$$

Consider the right hand side, $AB + AC$. We know AB from part (a) above. Then

$$\begin{aligned} AC &= \begin{pmatrix} 6 & 4 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} 8 & 3 \\ 0 & -3 \end{pmatrix} \\ &= \begin{pmatrix} 6(8)+4(0) & 6(3)+4(-3) \\ 2(8)+5(0) & 2(3)+5(-3) \end{pmatrix} = \begin{pmatrix} 48 & 6 \\ 16 & -9 \end{pmatrix} \end{aligned}$$

And therefore

$$\begin{aligned} AB + AC &= \begin{pmatrix} 2 & 34 \\ -14 & 15 \end{pmatrix} + \begin{pmatrix} 48 & 6 \\ 16 & -9 \end{pmatrix} \\ &= \begin{pmatrix} 50 & 40 \\ 2 & 6 \end{pmatrix} \end{aligned}$$

And we have shown that $A(B+C) = AB+AC$: namely the distributive law for matrix multplication over matrix addition.