MAU11404 Tutorial Sheet
Submit via Blackboard by 5pm, Friday 19th February 2021 [1]

1. For the following differential equations: state the order, type (ODE or PDE) and whether the equation is linear or non-linear.

   (a) $\alpha y' + \beta y = \gamma(x)$
   (b) $8y''' + xy = \cos(x)$
   (c) $\frac{d^2y}{dx^2} + e^x \frac{dy}{dx} = x \ln(x)$
   (d) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial t} = \frac{1}{2}(x^2 - t^2)$
   (e) $\alpha \frac{\partial^3 u}{\partial^2 x \partial t} = \beta + \frac{\partial u}{\partial y}$
   (f) $\frac{\partial^3 z}{\partial^2 x \partial t} + \frac{\partial z}{\partial t} - z^2 = 0$

   Note that α and β are constants in each of the equations above.

2. Show that $y(x) = Cx^{-\frac{3}{2}}$ is a solution of the differential equation,

$$4x^2 y'' + 12xy' + 3y = 0,\ x > 0,$$

   where C is an arbitrary constant.

   Use the *initial conditions*: $y(4) = \frac{1}{8}$ and $y'(4) = -\frac{3}{64}$, to show that in this case the unique solution to the differential equation above is $y(x) = x^{-\frac{3}{2}}$.

3. Use the integrating factor method obtain a general solution for

   (a) $(x+1)y' + y = (x+1)^2$
   (b) $y' + y\cot(x) = \cos(x)$
   (c) $\frac{dy}{dx} + 2xy = 2xe^{-x^2}$
   (d) $\cos(x)\frac{dy}{dx} + \sin(x)y = 2\cos^3(x)\sin(x) - 1$
   (e) $y' = 6x(y+1)$

4. Use the integrating factor method and the initial condition to find the solution, including the constant of integration, of

   (a) $xy' + 2y = 10x^2, \quad y(1) = 3$
   (b) $xy' - y = x^2, \quad y(1) = 3$
   (c) $y' = y\tan(x) - \sec(x), \quad y(0) = 1$

[1] also posted at http://www.maths.tcd.ie/˜ryan/11404.html