UNIVERSITY OF DUBLIN
TRINITY COLLEGE

FACULTY OF SCIENCE

SCHOOL OF MATHEMATICS

SS Mathematics **Trinity Term**
SS Theoretical Physics

COURSE 443

XXX XX XX GMB 9:30 - 12:30

Dr. S. Ryan

Answer
FOUR Questions from Section A
and
THREE Questions from Section B

Section A

1. Find the equation for the transition line $\frac{dP}{dT}$ (the Clausius-Clapyron equation).

   Comment on the information it yields about the nature of the phase transition.

2. Given
$$Z_\Omega = \sum_{N=0}^{\infty} \frac{1}{N!} \left(\frac{zV}{\lambda^3} \right)^N$$
   for a perfect, non-relativistic gas in classical statistical mechanics.

   Calculate the chemical potential, μ.

3. Write down Landau's expansion of the free energy $F(T, M, B)$ in terms of M and B.

   Determine $M(z)$ at equilibrium.

4. Sketch the Gibbs paradox in classical thermodynamics.

5. Show that $0 \leq z \leq 1$ for Bose-Einstein statistics. You may assume
$$N = A \int_0^\infty d\epsilon \epsilon^{1/2} n(\epsilon) \ \ , \ \ n(\epsilon) = \frac{1}{e^{\beta(\epsilon - \mu)} - 1}.$$

6. Discuss energy fluctuations in the classical canonical ensemble.

Section B

1. Use the canonical ensemble to derive the Van der Waals equation of state for a gass of N identical molecules of mass m in a volume, V at low density, with an intermolecular interaction potential

$$V(|\vec{x}|) = \begin{cases} 0 & |\vec{x}| > L_0 \\ -V_0 & a < |\vec{x}| < L_0 (V_0 small) \\ \infty & |\vec{x}| < a \end{cases}$$

where $|\vec{x}|$ is the distance between two molecules.

2. Consider a perfect non-relativistic gas in classical statistical mechanics. Calculate the grand canonical potential $\Omega = -PV$.

   Obtain the equation for hte equilibrium concentrations $c_i = N_i/N$ for the reaction $2H_2 + O_2 \leftrightarrow 2H_2$ at fixed pressure and temperature, treating the constituents as a perfect gas.

3. Consider the d-dimensional Ising model with Hamiltonian

$$H = \frac{J}{2} \sum_i \sum_{j=n.n} S_i S_j - \mathcal{H} \sum_i S_i, \, J < 0$$

   where $S_i = \pm 1/2$ and $n.n$ indicates the sum over nearest neighbours and $\mathcal{H}$ is an external magnetic field.

   Compute the critical temperature for spontaneous magnetisation in the Mean Field Approximation.

   Discuss the behaviour of the magnetic susceptibility χ near the critical temperature T_c and determine the corresponding critical exponent.

4. The internal energy $U(T,V)$ for a gas of photons (black body radiation) is given by

$$U(T,V) = V \int_0^\infty d\omega u(\omega, T)$$

   Find the function $u(\omega, T)$ and sketch the function for fixed T.

   Compute the entropy density, $\frac{S}{V}$ as a function of T

5. Consider a gas of N noninteraction identical fermions in a box of volume V. You may assume

$$N = A\int \frac{\sqrt{E}dE}{e^{\beta(E-\mu)}+1} \quad A = (2S+1)m^{3/2}\frac{4\pi V\sqrt{2}}{(2\pi\hbar)^3}$$

and

$$\Omega = -\frac{2}{3}A\int \frac{E^{3/2}dE}{e^{\beta(E-\mu)}+1}$$

where $\Omega = -PV$ is the grand canonical ensemble.

(a) Compute the zero point pressure as a function of N/V.

(b) Show that the specific hear c_V vanishes as $aT \to 0$.