## Example

The exact value of the integral

$$\int_0^{\pi/2} 5(e^\pi - 2)^{-1} e^{2x} \cos x dx$$

is 1.

Using the polynomial extrapolation method of Romberg and carrying 12 digits determine $T_{ik}$ and $U_{ik}$ for $0 \leq i \leq 6$ and $0 \leq k \leq 3$.

## Solution

| $i$ | $T_{io}$ | $T_{i1}$ | $T_{i2}$ | $T_{i3}$ |
|---|---|---|---|---|
| 0 | 0.185 755 068924 | | | |
| 1 | 0.724 727 335089 | 0.904 384 757 145 | | |
| 2 | 0.925 565 035158 | 0.992 519 935 182 | 0.998 386 013717 | |
| 3 | 0.981 021 630069 | 0.999 507 161 706 | 0.999 973 576808 | 0.999 998 776222 |
| 4 | 0.995 232 017388 | 0.999 968 813 161 | 0.999 999 589925 | 1.000 000 00283 |
| 5 | 0.998 806 537974 | 0.999 998 044 836 | 0.999 999 993614 | 1.000 000 00002 |
| 6 | 0.999 701 542775 | 0.999 999 877 709 | 0.999 999 999901 | 1.000 000 00000 |
| $i$ | $U_{io}$ | $U_{i1}$ | $U_{i2}$ | $U_{i3}$ |
| 0 | 1.263 699 60126 | | | |
| 1 | 1.126 402 73523 | 1.080 637 11322 | | |
| 2 | 1.036 478 22498 | 1.006 503 38823 | 1.001 561 13990 | |
| 3 | 1.009 442 40471 | 1.000 430 46462 | 1.000 025 60304 | 1.000 001 22944 |
| 4 | 1.002 381 05856 | 1.000 027 27651 | 1.000 000 39730 | 0.999 999 997211 |
| 5 | 1.000 596 54758 | 1.000 001 71058 | 1.000 000 00619 | 0.999 999 999978 |
| 6 | 1.000 149 21714 | 1.000 000 10700 | 1.000 000 00009 | 1.000 000 00000 |