Example

The exact value of the integral

$$\int_0^{\pi/2} 5(e^{\pi} - 2)^{-1} e^{2x} \cos x dx$$

is 1.

Using the polynomial extrapolation method of Romberg and carrying 12 digits determine T_{ik} and U_{ik} for $0 \leq i \leq 6$ and $0 \leq k \leq 3$.

Solution

i	T_{io}	T_{i1}	T_{i2}	T_{i3}
0	0.185 755 068924			
1	0.724 727 335089	0.904 384 757 145		
2	0.925 565 035158	0.992 519 935 182	0.998 386 013717	
3	0.981 021 630069	0.999 507 161 706	0.999 973 576808	0.999 998 776222
4	0.995 232 017388	0.999 968 813 161	0.999 999 589925	1.000 000 00283
5	0.998 806 537974	0.999 998 044 836	0.999 999 993614	1.000 000 00002
6	0.999 701 542775	0.999 999 877 709	0.999 999 999901	1.000 000 00000
i	U_{io}	U_{i1}	U_{i2}	U_{i3}
0	1.263 699 60126			
1	1.126 402 73523	1.080 637 11322		
2	1.036 478 22498	1.006 503 38823	1.001 561 13990	
3	1.009 442 40471	1.000 430 46462	1.000 025 60304	1.000 001 22944
4	1.002 381 05856	1.000 027 27651	1.000 000 39730	0.999 999 997211
5	1.000 596 54758	1.000 001 71058	1.000 000 00619	0.999 999 999978
6	1.000 149 21714	1.000 000 10700	1.000 000 00009	1.000 000 00000