UNIVERSITY OF DUBLIN

MA261

TRINITY COLLEGE

Faculty of Science

school of mathematics

SF Mathematics **Sample Paper 2001**

Course 261

Someday, Some date, 2001 Someplace Sometime

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Attempt XX Questions

1. Given the 3×3 matrix A and the vector, b

$$A = \begin{pmatrix} 1 & 0 & 5 \\ 1 & 1 & 1 \\ 2 & 1 & 4 \end{pmatrix} \quad , \quad b = \begin{pmatrix} 21 \\ 16 \\ 31 \end{pmatrix}$$

Use Gauss-Jordan elemination to find the matrix inverse, A^{-1}. Solve the system $Ax = b$ for the vector x.
Determine the condition number, $c_\infty(A)$ and comment on whether or not the matrix A is well-conditioned.

2. Answer 4 of the 6 parts, *(a)* to *(d)*:

(a) Describe the Jacobi and Gauss-Seidel techniques to solve a matrix equation $Ax = b$ for x.

(b) Prove the Jacobi (or Gauss-Seidel) method is convergent if the matrix A is Strictly Diagonally Dominant (SDD).

(c) Determine if the matrix, A, in the question 1 is SDD.

(d) For either the Jacobi or the Gauss-Seidel Method, sketch, in pseudo-code, one step of the numerical algorithm to implement the scheme.

(e) Describe Krylov methods and how they differ from Jacobi/GS.

(f) Show that if R and upper triangular matrix and C an upper Hessenberg matrix, then RCR^{-1} is also upper Hessenberg.

3. Prove that for $n+1$ arbitrary points, (x_i, f_i), $i = 0, 1, \ldots, n$ and $x_i \neq x_k$ there exists a unique polynomial $P \in \Pi_n$ with $P(x_i) = f_i$.

   Given the data

x_i	0	1	3
f_i	1	3	2

   Use Neville's interpolating algorithm to determine $P\left(\frac{2}{3}\right)$.

   Use Newton's divided difference algorithm to determine the form of the interpolating polynomial and also $P(2)$.

4. Prove that Simpson's Rule is an order 4 method.

   Determine the Peano kernel, $K(t)$ for Simpson's rule and $n = 2$ (not $n = 3$) in $[-1, +1]$. Does it change signin the interval of integration?