Course 161/2S3, Tutorial, Hilary Term, 2006

- Consider the second order differential equation

$$\frac{d^2x}{dt^2}(t) = -x(t)$$

with initial conditions $x(0) = 1$ and $\frac{dx}{dt}(0) = 0$. Write this as two coupled first order differential equations.

Apply the Euler algoritm to the derived equations for 4 integration steps, each of size $h = 0.1$ to find $x(t = 0.4)$.

- The trapezoidal rule approximates

$$I = \int_x^{x+h} f(x)dx$$

with the formula

$$I = \frac{h}{2}\left(f(x) + f(x+h)\right)$$

Show that the error of this approximation is $O(h^3)$.

The n step extended trapezoidal rule divides the interval from a to b into n equal subintervals, and uses the trapezoidal rule to evaluate the integral on each subinterval. Estimate the error made in approximating

$$\int_a^b dx f(x)$$

with this extended rule. This error estimate should be given in terms of $b - a$ and n.

Evaluate

$$\int_0^2 \frac{\sin(x)}{x+1}dx$$

with $h = 0.5$ using trapezoidal rule.

[partly from 1999 Summer paper]

- Apply the Newton-Raphson method to find the roots of

$$f(x) = x^2 - 3x - 7$$

correct to three decimal places.