UNIVERSITY OF DUBLIN

TRINITY COLLEGE

FACULTY OF SCIENCE

SCHOOL OF MATHEMATICS

JF Mathematics **Michaelmas Term 2006**
JF Theoretical Physics
JF Two Subject Mod

COURSE 111

Wednesday, December 13 Regent House 09.30 — 11.30

Dr. S. Ryan

ATTEMPT FOUR QUESTIONS

Log tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used.

Note: This is a sample paper for the Christmas exam so you can see the format and style of questions you might be asked.

The questions here are drawn from your tutorial questions and class notes. You shouldn't interpret this paper as an indication of what material may, or may not, appear on the exam.

1. Define what is meant by an equivalence relation, $\sim$, on a set A, making reference to reflexivity, symmetry and transitivity.

   Define equivalence classes.

   Consider the set S of all integers. Given $a, b \in S$ define $a \sim b$ if $a - b$ is an even integer. Show that this defines an equivalence relation and determine the equivalence classes of this relation.

2. Define a bijective map and give an example of a bijective map, g, from the set of real numbers, R to itself.

   Consider the map $\sigma : R \to R$ defined by $\sigma(x) = x^3 - x$. This is not a bijection. State why not.

   Consider two maps from a set S to a set T and from the set T to a set U, $\sigma : S \to T$ and $\tau : T \to U$. Prove that $\sigma \circ \tau$ is one-to-one if each of σ and τ is one-to-one.

3. Consider a nonempty set of elements, labelled G. List the group axioms for G to form a group.

   Suppose G is a cyclic group of order 4, written $G = \{e, a, a^2, a^3\}$ and $a^4 = e$. Find the order of each element of G. Find all the subgroups of G.

   List the distinct left cosets of the subgroup $H = \{e, a^2\}$ in G.

4. Given a nonempty subset H of a group G. When does H form a subgroup of G?

   Prove that for H and K both subgroups of a group G. The product, HK is itself a subgroup of G iff $HK = KH$.

   Verify that $H = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} \in G \;\middle|\; ad \neq 0 \right\}$ is a subgroup of the group of 2×2 matrices expressed as, $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \;\middle|\; ad - bc \neq 0 \right\}$ under matrix multiplication.