UNIVERSITY OF DUBLIN

XMA1111

TRINITY COLLEGE

Faculty of Science

school of mathematics

JF Mathematics
JF Theoretical Physics
JF Two Subject Mod

Michaelmas Term 2006

Course 111

Wednesday, December 13 | Regent House | 09.30 — 11.30

Dr. S. Ryan

ATTEMPT FOUR QUESTIONS

Log tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used.

1. Consider three sets, A, B and C. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

   Consider a map, σ from a set A to a set B and a map τ from set B to set C. Prove that the composition of maps, $\sigma \circ \tau$ is one-to-one if each of σ and τ is one-to-one.

   Consider a map $\phi : R \to R$, from the set of real numbers to itself, defined by $\phi(x) = e^x$. State, giving a reason for your answer, if this map is a bijection?

2. Consider a nonempty set of elements forming a group, G. Prove that every element $a \in G$ has a unique inverse in G.

   Consider the symmetry group of an equilateral triangle in the plane. List the 6 rigid transformations that leave the triangle unchanged and show, by constructing a Cayley table, that these 6 motions form a group.

   Prove that a nonempty subset, H of a group G is a subgroup if and only if (i) $a, b \in H \Rightarrow ab \in H$ and (ii) $a \in H \Rightarrow a^{-1} \in H$.

   From the list of symmetries of the triangle, consider only the rotations which leave it invariant and show that this subset of symmetries forms a subgroup of the full symmetry group, whose Cayley table you determined above.

3. A subgroup, N of a group G is normal if for every $g \in G$ and $n \in N$, $gng^{-1} \in N$.

   Prove that N is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.

   Consider the cyclic group, $G = \{e, g, g^2, g^3\}$ under addition modulo 4 (where $g^4 = e$, the group identity). Determine the proper subgroups of G and the left and right cosets of $H = \{e, g^2\}$ in G.

   Show that $G|H$ forms a group.

4. Prove that if a map ϕ is a homomorphism between two groups, G and $\bar{G}$ then (i) $\phi(e) = (\bar{e})$ and (ii) $\phi(x^{-1}) = \phi(x)^{-1}, \forall x \in G$, where e and $\bar{e}$ are the identity elements in the groups G and $\bar{G}$ respectively.

Given the set of integers, Z, which forms a group under addition and the elements $\{+1, -1\}$ which form a group under multiplication, and a map $\phi(a) : Z \rightarrow \{+1, -1\}$ defined by $\phi(a) = -1^a$. Show that ϕ is a homomorphism and determine the kernel of the map.

Using this information explain why ϕ is not an isomorphism.