Lattice QCD: methods and phenomenology for hadron spectroscopy

Sinéad M. Ryan
Trinity College Dublin

Methods of Effective Field Theory and Lattice Field Theory, TUM, July 2017

Housekeeping

- Web page for these lectures at http://www.maths.tcd.ie/~ryan/TUM2017
- Lectures, homework problems and data as well as references will all be there (if not this morning then very soon!).

Plan

- **Lecture 1 and 2:**
  - Motivation - why is hadron spectroscopy interesting?
  - Short discussion of the quark model
  - Lattice QCD - introduction/recap and benchmark results
  - New(ish) ideas for going beyond ground states
  - Phenomenology
    - the physics of single hadrons
    - above thresholds - scattering and resonances
- **Lecture 3:**
  - Reminder of exotic quantum numbers and other exotica including hybrids
  - The XYZ states in charmonium
  - Review of lattice results
    - coupled channel systems
    - focus on heavy quark systems and the particular challenges for lattice
- Summary and outlook

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Aim: an introduction to hadronic physics from lattice QCD - to understand methods, the difficulties and the results and errors in a calculation.

- Note - I draw on lots of information from other talks at this school

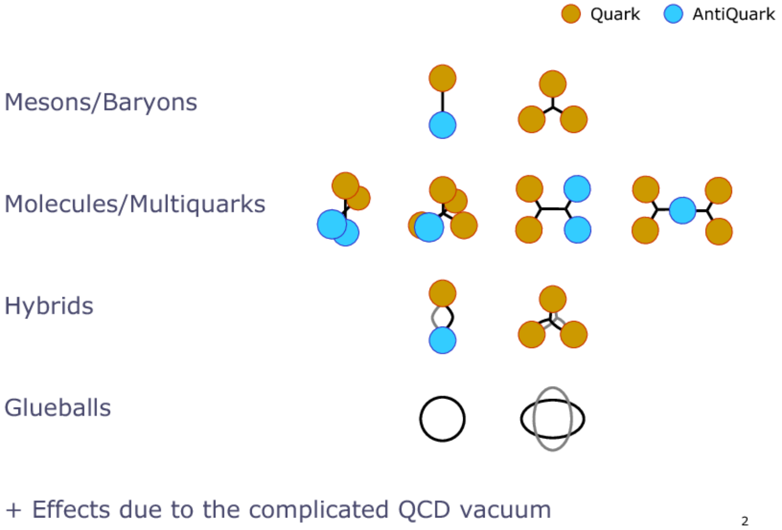
Introduction

There are many interesting questions in hadronic and nuclear physics that we would like to answer from first principles:

- What is the nature of the X,Y,Z states in charmonium and bottomonium?
- What is the dynamics and structure of exotic and hybrid states?
- How sensitive are "fine-tuned" quantities like $M_n - M_p$ to the values of fundamental parameters?
- What is the equation of state of dense nuclear matter in neutron stars?
- Can we test/break the Standard Model at low energies
    - relevant since SM complete with Higgs
    - no significant BSM physics observed other than $g-2$ and proton radius.
    - Can we determine reliable inputs for Dark Matter searches?

Not an exhaustive list of course!

Objects of interest - in hadron spectroscopy



A constituent model

- QCD has fundamental objects: quarks (in 6 flavours) and gluons
- Fields of the lagrangian are combined in colorless combinations: the mesons and baryons. Confinement.

quark model object	structure
meson	$3 \otimes \bar{3} = 1 \oplus 8$
baryon	$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$
hybrid	$\bar{3} \otimes 8 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 8 \oplus 10 \oplus 10$
glueball	$8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus 10$
⋮	⋮

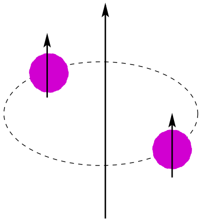
- This is a model. QCD does not always respect this constituent picture! There can be strong mixing.

Classifying states: mesons

- Recall that continuum states are classified by J^{PC} multiplets (representations of the Poincaré symmetry):
- Recall the naming scheme: $n^{2S+1}L_J$ with $S = \{0, 1\}$ and $L = \{0, 1, \ldots\}$
- J, hadron angular momentum, $|L - S| \leq J \leq |L + S|$
- $P = (-1)^{(L+1)}$, parity
- $C = (-1)^{(L+S)}$, charge conjugation. Only for $q\bar{q}$ states of same quark and antiquark flavour. So, not a good quantum number for eg heavy-light mesons $(D_{(s)}, B_{(s)})$.

Mesons

- two spin-half fermions ${}^{2S+1}L_J$
- $S = 0$ for antiparallel quark spins and $S = 1$ for parallel quark spins;



- States in the natural spin-parity series have $P = (-1)^J$ then $S = 1$ and $CP = +1$:
  - $J^{PC} = 0^{-+}, 0^{++}, 1^{--}, 1^{+-}, 2^{--}, 2^{-+}, \ldots$ **allowed**
- States with $P = (-1)^J$ but $CP = -1$ **forbidden** in $q\bar{q}$ model of mesons:

- $J^{PC} = 0^{+-}, 0^{--}, 1^{-+}, 2^{+-}, 3^{-+}, \ldots$ forbidden (by quark model rules)
- These are **EXOTIC** states: not just a $q\bar{q}$ pair ... more on Friday

Baryons

Baryon number $B = 1$: three quarks in colourless combination

- J is half-integer, C not a good quantum number: states classified by J^P
- Spin-statistics: a baryon wavefunction must be antisymmetric under exchange of any 2 quarks.
- Totally antisymmetric combinations of the colour indices of 3 quarks
- The remaining labels: flavour, spin and spatial structure must be in totally symmetric combinations

$$|qqq\rangle_A = |\text{color}\rangle_A \times |\text{space, spin, flavour}\rangle_S$$

With three flavours, the decomposition in flavour is

$$3 \otimes 3 \otimes 3 = 10_S \oplus 8_M \oplus 8_M \oplus 1_A$$

Many more states predicted than observed: missing resonance problem

Why Lattice QCD ?

- A systematically-improvable non-perturbative formulation of QCD
  - Well-defined theory with the lattice a UV regulator
- Arbitrary precision is in principle possible
  - of course algorithmic and field-theoretic "wrinkles" can make this challenging!
- Starts from first principles - i.e. from the QCD Lagrangian
  - inputs are quark mass(es) and the coupling - can explore mass dependence and coupling dependence but getting to physical values can be hard!
  - in principle can calculate inputs for nuclear many-body calculations with better accuracy than experimental measurements. Starting from nucleon-nucleon phase shifts and on to hyperon-nucleon (YN) etc.

A typical road map

- Develop methods and verify calculations through precision comparison with lattice and with experiment.
- Make predictions - subsequently verified experimentally.
- Make robust, precise calculations of quantities beyond the reach of experiment.

A potted history

- **1974** Lattice QCD formulated by K.G. Wilson
- **1980** Numerical Monte Carlo calculations by M. Creutz
- **1989** "and extraordinary increase in computing power (10^8 is I think not enough) and equally powerful algorithmic advances will be necessary before a full interaction with experiment takes place." Wilson @ Lattice Conference in Capri.
- **Now** at 100 *TFlops* — 1 *PFlop*
- Lattice QCD contributes to development of computing QCDSP - QCDOC - BlueGene.

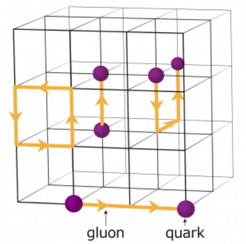
Learning from history ...
better computers help but better ideaas are crucial!
that's what we will focus on ...

A lattice QCD primer

Start from the QCD Lagrangian:

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

- Gluon fields are SU(3) matrices - links of a hypercube.
  $A_\mu(x) \rightarrow U(x,\mu) = e^{-iagA^b_\mu(x)t^b}$
- Quark fields $\psi(x)$ on sites with color, flavor, Dirac indices.
  Fermion discretisation - Wilson, Staggered, Overlap.
- Derivatives $\rightarrow$ finite differences:
  $\nabla^{\mathrm{fwd}}_\mu \psi(x) = \frac{1}{a}\left[U_\mu(x)\psi(x+a\hat{\mu}) - \psi(x)\right]$



Solve the QCD path integral on a finite lattice with spacing $a \neq 0$ estimated stochastically by Monte Carlo. Can only be done effectively in a Euclidean space-time metric (no useful importance sampling weight for the theory in Minkowski space).

Observables determined from (Euclidean) path integrals of the QCD action

$$\langle \mathcal{O} \rangle = 1/Z \int \mathcal{D}U \mathcal{D}\bar{\psi}\mathcal{D}\psi \; \mathcal{O}[U,\bar{\psi},\psi] e^{-S[U,\bar{\psi},\psi]}$$

Correlators in a Lattice Euclidean Field Theory I

- Physical observables $\mathcal{O}$ is determined from

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\Psi \mathcal{D}\bar{\Psi} \mathcal{O} e^{-S_{QCD}}$$

- Analytically integrate Grassman fields $(\Psi, \bar{\Psi}) \rightarrow$ factors of $\det M$ the fermion mx.

$$\langle \mathcal{O} \rangle \overset{N_f=2}{=} \frac{1}{Z} \int \mathcal{D}U \det M^2 \mathcal{O} e^{-S_G}$$

  The expectation value is calculated by importance sampling of gauge fields and averaging over ensembles.

- Simulate N_{cfg} samples of the field configuration, then

$$\langle \mathcal{O} \rangle = \lim_{N_{cfg} \rightarrow \infty} \frac{1}{N_cfg} \sum_{i=1}^{N_cfg} \mathcal{O}_i[U_i]$$

- At N_{cfg} finite correlation functions have a (improvable!) statistically uncertainty $\sim 1/\sqrt{N_{cfg}}$.
- Calculating $\det M$ for M a large, sparse matrix with small eigenvalues takes $> 80\%$ of compute cycles in configuration generation. $\det M = 1$ is the quenched approximation.

Correlators in lattice field theory II

- We are interested in two-point correlation functions built from interpolating operators (functions of Ψ):
  - Eg the local meson operator $\mathcal{O}(x) = \bar{\Psi}_a(x)\Gamma\Psi_b(x)$
  - Γ an element of the Dirac algebra with possible displacements; a and b flavour indices
- The two-point function is then

$$C(t) = \langle \mathcal{O}(x)\mathcal{O}^\dagger(0)\rangle = \langle \bar{\Psi}_a(x)\Gamma\Psi_b(x)\bar{\Psi}_b(0)\Gamma^\dagger\Psi_a(0)\rangle$$

  where $x \equiv (t, \mathbf{x})$; $t \geq 0$
- Using Wick's theorem to contract quark fields replaces fields $\rightarrow$ quark propagators

$$\begin{aligned} C(t, \mathbf{x}) &= -\langle \mathrm{Tr}(M_a^{-1}(0, x)\Gamma M^{-1}{}_b(x, 0)\Gamma^\dagger)\rangle \\ &\quad +\delta_{ab}\langle \mathrm{Tr}(\Gamma M^{-1}{}_a(x, x))\mathrm{Tr}(\Gamma^\dagger M^{-1}{}_a(0, 0))\rangle \end{aligned}$$

  where the trace is over spin and colour.
- For flavour non-singlets ($a \neq b$) this leads to **[See specific example in homeworks]**

$$C(t, \mathbf{x}) = \langle \mathrm{Tr}(\gamma_5 M_a^{-1}(x, 0)^\dagger \gamma_5 \Gamma M_b^{-1}(x, 0)\Gamma^\dagger)\rangle$$

- We consider the correlation function in momentum space at zero momentum

$$C(\vec{p}, t) = \int d^3x e^{i\vec{p}\cdot\vec{x}} C(\vec{x}, t, 0, 0) \text{ and } C(0, t) = C(t) \sim \sum_{\vec{x}} C(\vec{x}, t, 0, 0)$$

Aside on Wick's theorem

We used Wick's theorem to contract quark fields and replace with propagators ...

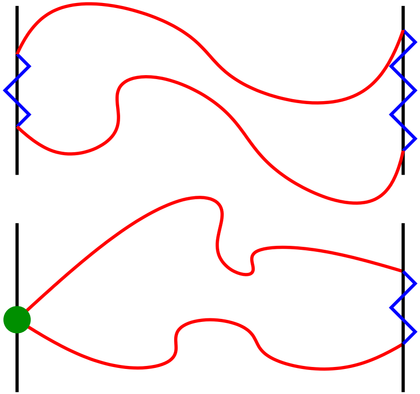
- Example — four field insertions: $\langle \psi_i \bar{\psi}_j \psi_k \bar{\psi}_l \rangle$
- The pairwise contraction can be done in two ways: $\psi_i \bar{\psi}_j \psi_k \bar{\psi}_l$ and $\psi_i \bar{\psi}_j \psi_k \bar{\psi}_l$
- Giving the propagator combination $M^{-1}_{ij} M^{-1}_{kl} - M^{-1}_{jk} M^{-1}_{il}$
- Minus-sign from the anti-commutation in second term.
- More fields means more combinations. Important in (eg.) isoscalar meson spectroscopy. We will see this again later.

Notes

- Fermions in lagrangian: sea quarks → fermion determinant
- Fermions in measurement: valence quarks → propagators
- The integral over gauge fields is done using importance sampling.
- γ_5 hermiticity: $M^{-1}(x, y) = \gamma_5 M^{-1}(y, x)^{\dagger}\gamma_5$ allows us to rewrite the correlator in terms of propagators from origin to all sites. Point (to-all) propagators
- Practically: $M(x, 0 : U)^{-1}$ compute a singe column (in space-time indices) with linear solvers
- For flavour singlets $a = b$ terms like $M^{-1}(x, x)$ - require the inverse of the **full** fermion matrix on each configuration. More on this later.

Come back later to the costs in a lattice calculation

PROPAGATOR CARTOON



The most general operator.

A restricted correlation function accessible to one point-to-all computation.

Saves compute time but doesn't use all information in the correlator. Precludes disconnected contributions ie flavour singlets.

A recipe for (meson) spectroscopy

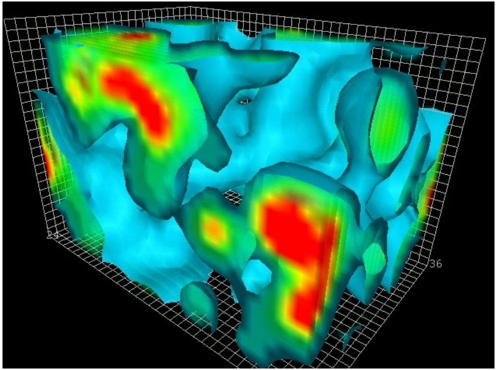
- Construct a basis of local and non-local operators $\bar{\Psi}(x)\Gamma D_i D_j \ldots \Psi(x)$ e.g. from *distilled* fields M. Peardon on Friday and [PRD80 (2009) 054506].
- Build a correlation matrix of two-point functions (from Lang & Peardon lectures):

$$C_{ij} = \langle 0|\mathcal{O}_i \mathcal{O}_j^\dagger|0\rangle = \sum_n \frac{Z_i^n Z_j^{n\dagger}}{2E_n} e^{-E_n t}$$

- Ground state mass from fits to $e^{-E_n t}$
  [Related homework problem and data on webpage]
- Beyond ground state: Solve generalised eigenvalue problem - Variational Approach
  $C_{ij}(t) v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)}$

- Eigenvalues: $\lambda^{(n)}(t) \sim e^{-E_n t}\left[1 + O(e^{-\Delta E t})\right]$ - principal correlator.
- Eigenvectors: related to overlaps: $Z_i^{(n)} = \sqrt{2E_n} e^{E_n t_0/2} v_j^{(n)\dagger} C_{ji}(t_0)$.

Typical size of a lattice calculation



[from D.Leinweber]

There are 2 compute intensive steps:

1. Generating Configurations - snapshots of the QCD vacuum

Volume: $32^3 \times 256$ (sites) $U_\mu(x)$ defined by $4 \times 8 \times 32^3 \times 256$ real numbers

2. Quark Propagation

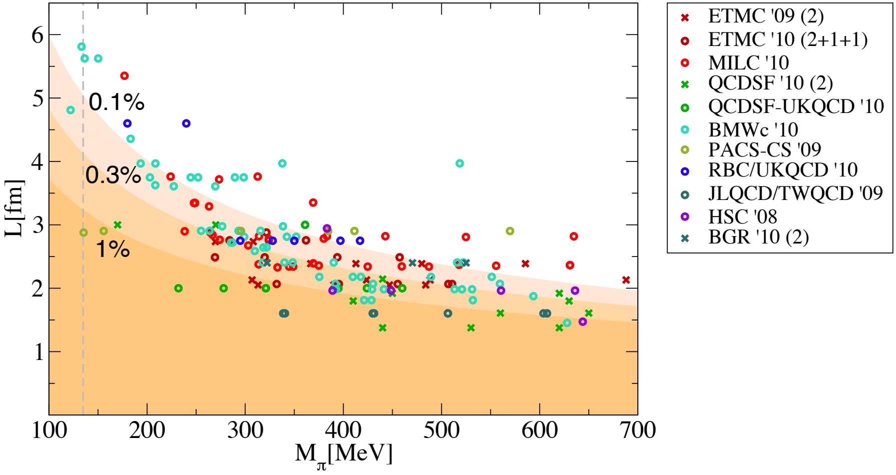
Volume: $32^3 \times 256$ (sites) $\rightarrow M$ is a 100 million x 100 million sparse matrix with complex entries.

Solving QCD requires supercomputing resources worldwide.

Validation:
can we reproduce known results and make verified predictions?

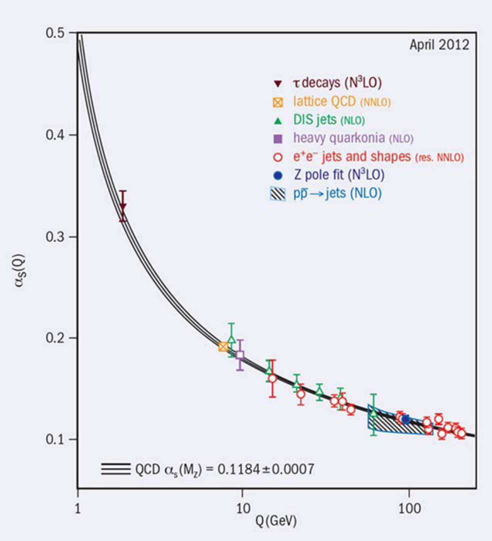
How are we doing?

C. Hoelbling, arXiV.1102.0410 [already 6yrs old]

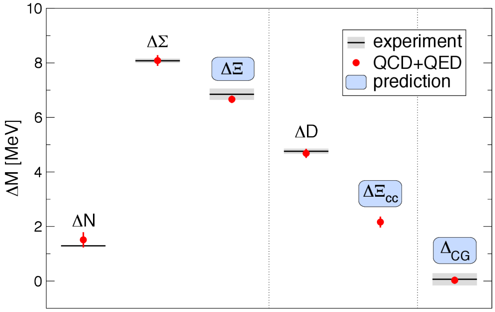


Validation

The running coupling, α_s



Baryon electromagnetic mass splittings



QED + QCD

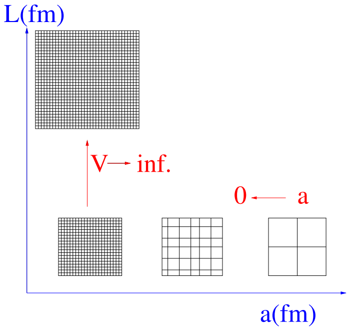
BMW Collab. Science 347 (2015) 1452

Further thoughts on lattice calculations

Compromises and the Consequences

1. Working in a finite box at finite grid spacing

- Identify a "scaling window" where physics doesn't change/changes weakly/in a well-defined way with a or V. Recover continuum **QCD** by extrapolation.
- Lattice spacing small enough to resolve structures induced by strong dynamics.
- Volume large enough to contain lightest particle in spectrum: $m_\pi L \geq 2\pi$.



A costly procedure but a regular feature in lattice calculations now

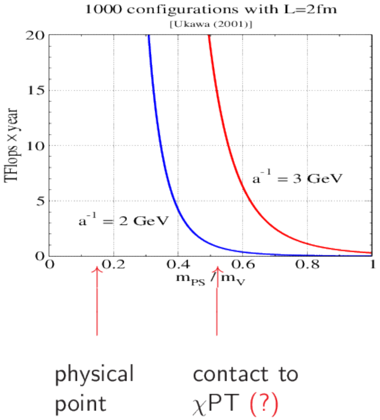
2. Simulating at physical quark masses: light quarks

- Light quarks in gauge generation through fermion determinant M.
- Computational cost grows rapidly with decreasing quark mass $\rightarrow m_q = m_{u,d}$ costly.

- The Berlin wall - from Lattice 2001

$$C \propto \left(\frac{m_\pi}{m_\rho}\right)^{-6} (L)^5 (a)^{-7}$$

- Work horse is Hybrid Monte Carlo (HMC) Duane et al 1987
- Many improvements over the years for all fermion discretisations



- The wall has come down - Physical point can be reached!
- Caution: care needed vis location of decay thresholds and identification of resonances.

2. Simulating at physical quark masses: heavy quarks

- See Andreas Kronfeld's lectures from last week.
- Discretisation errors grow as $\mathcal{O}(am_q)$ becoming large for reasonable a and heavy quarks
- Bottom quarks treated with Effective Field Theories - NRQCD, Fermilab etc
  - Continuum limits and EFTs can be tricky - not always possible e.g. with NRQCD
  - Controlling systematics important for precision CKM physics
- Charm quarks can be handled relativistically
  - Anisotropic lattices useful here: $a_s \neq a_t$ and $a_t m_c < 1$. Care needed to ensure errors $\mathcal{O}(a_s m_c)$ are not large.

Better algorithms for physical light quarks and/or chiral extrapolation. Relativistic m_b is in reach.

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Turn a "weakness" into a strength by using lattice simulations to study *quark mass dependence*!

3. Breaking symmetry

- Almost all symmetries of **QCD** are preserved. But Lorentz symmetry broken at $a \neq 0$ so $SO(4)$ rotation group broken to discrete rotation group of a hypercube.
- Angular momentum and parity J^P correspond to irreducible representations of the rotation group $O(3)$
- A spatially isotropic lattice breaks $O(3) \rightarrow O_h$, the cubic point group.
- Eigenstates of the lattice $\mathcal{H}$ transform under irreps of O_h so states are classified by these irreps and not by J^P.
- Classify states by irreps of O_h and relate by subduction to J values of O_3.
- 5 irreps of $O(3)$ and an infinite number for J^P so values are distributed across lattice irreps.
- Lots of degeneracies in subduction for $J \geq 2$ and physical near-degeneracies. Complicates spin identification.

Connecting lattice and continuum groups

	A_1	A_2	E	T_1	T_2
$J=0$	x				
$J=1$				x	
$J=2$			x		x
$J=3$		x		x	x
$J=4$	x		x	x	x
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

- In principle, to identify a $J=2$ state, results from E and T_2 at finite a should extrapolate to the same result.
- An expensive business(!)
- Even then, is there enough information to disentangle high-spin states e.g. $4 = 0 \oplus 1 \oplus 2$?
- In charmonium a radial excitation of the near-degenerate $(0^{++}, 1^{++}, 2^{++})$ could be close in energy to the 4^{++} ground state.

A solution - Spin identification at finite lattice spacing: 0707.4162, 1204.5425

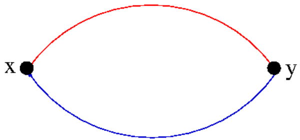
4. Making propagators - Wick revisited

- Recall that to calculate particle properties the fermion matrix M - a large ($\sim 10^8 \times 10^8$) matrix is inverted followed by Wick contractions of quark fields.

$$C(t,\mathbf{x}) = \langle \mathrm{Tr}(\gamma_5 M_a^{-1}(x,0)^{\dagger}\gamma_5 \Gamma M_b^{-1}(x,0)\Gamma^{\dagger})\rangle$$

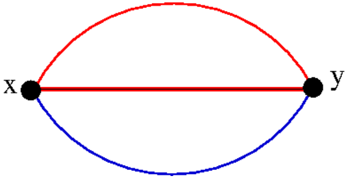
Mesons e.g. a pion

$$\langle \bar{u}(x)\gamma_5 d(x)\bar{d}(y)\gamma_5 u(y)\rangle = \mathrm{Tr}\{\gamma_5 M_{dd}^{-1}(x,y;U)^{\dagger}\gamma_5 M_{uu}^{-1}(y,x;U)\Gamma^{\dagger})\}$$



Baryons e.g. a proton:

2 up quarks, 1 down quark. Must keep track of which u quark from x goes to which u quark at y - 2 contractions: $(N_u! \times N_d!)$
Now, for proton-proton scattering have $4! \times 2! = 48$ contractions
For He3 (ppn) $5! \times 4! = 2880$ and for He4 (pppn) $6! \times 6! = 518400$.



Symmetries reduce the problem and better algorithms

Yamazaki et al, PRD81 2010; Orginos& Detmold PRD87 2013

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5. Making propagators - signal-to-noise

- Particularly severe for nucleons.
- Signal for a proton correlation function $\sim Ze^{-m_N t}\left[1+\delta Z_n e^{-(E_n - m_N)t}\right]$.
- Signal to noise for a proton is then $\sim \sqrt{N_{cfg}} e^{-(m_N - \frac{3}{2} m_\pi)t}$ with $m_N \sim 939 MeV$ and $m_\pi \sim 135 MeV$.
- Signal to noise for A nucleons is $\sim \sqrt{N_{cfg}} e^{-A(m_N - \frac{3}{2} m_\pi)t}$

To solve this extract information from the correlators at early Euclidean time i.e. before the noise becomes dominant. Requires "better" operators for nucleons (which also increases the Wick contraction problem!).

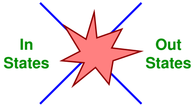
6. Setting the scale

Lattice quantities are computed in lattice units e.g. am_N. Convert to physical units to compare to experiment/make predictions of physical observables e.g. masses and form factors.

Choose an observable O that is relatively easy to calculate and insensitive to e.g. up and down quark masses (which may not be correct in the simulation) and match to its experimental value to determine a. This quantity is no longer a prediction!

- Stable masses e.g. M_ω, M_K
- Force between static quarks: $F(r) = -\frac{B}{r^2} + \sigma$ and $r_0^2 F(r_0) = 1.65$ with $r_0 \sim 0.5 fm$ from experiment/phenomenology of charmonium bottomonium systems.
- Wilson flow. A new idea that is precisely and easily calculated from the gauge action.

Many reasonable choices and discretisation errors mean there is some uncertainty from this procedure.



7. Working in Euclidean time.

- Scattering matrix elements not directly accessible from Euclidean QFT [*Maiani-Testa theorem*]. Scattering matrix elements: asymptotic $|\text{in}\rangle$, $|\text{out}\rangle$ states: $\langle\text{out}|e^{i\hat{H}t}|\text{in}\rangle \rightarrow \langle\text{out}|e^{-\hat{H}t}|\text{in}\rangle$. Euclidean metric: project onto ground state. Analytic continuation of numerical correlators an ill-posed problem.

Lüscher and generalisations of: method for indirect access. See more on this later.

8. Quenching

- A computational expedient to set $\det M = 1$ in gauge configuration generation.
- Rarely necessary now, results in a non-unitary theory so not a good approximation of nature.
- Sometimes useful for investigating new methods.

No longer an issue: Simulations with $N_f = 2, 2+1, 2+1+1$.

Two strategies for progress in hadron phenomenology

In lattice calculations there are complementary efforts

“Gold-plated” quantities

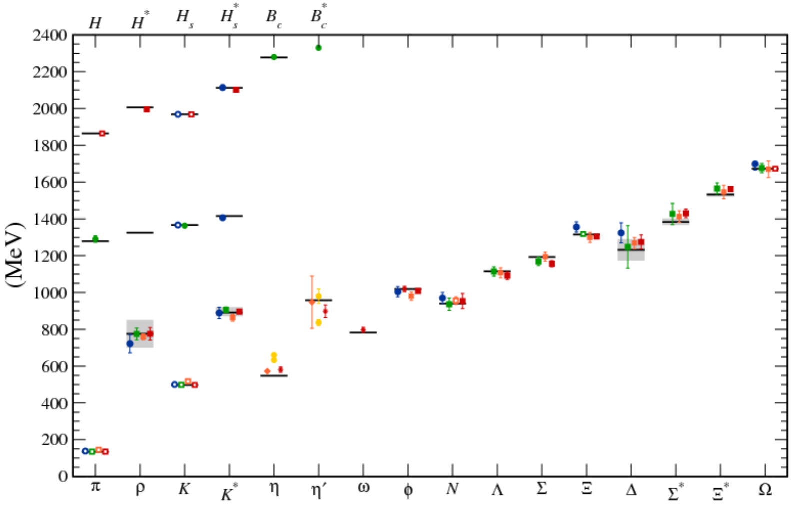
- includes single hadron states, or decays below thresholds
- phenomenologically relevant
- incremental progress
- robust/well-tested methods
- careful error budgeting

New directions

- new ideas - theoretical and algorithmic that open new avenues
- recent examples are scattering states, nuclear physics, g-2, ...
- also improves gold-plated quantities
- pioneering - error budgets not yet “robust”

- Look at “gold-plated” results first
- Review new ideas for pioneering calculations next.

Strategies for progress: gold plated quantities - a selection



A. Kronfeld, Ann.Rev.Nucl.Part.Sci. 62 (2012)

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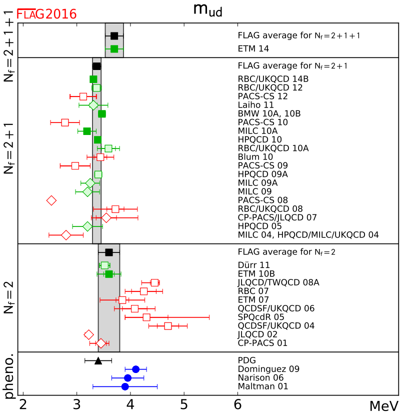
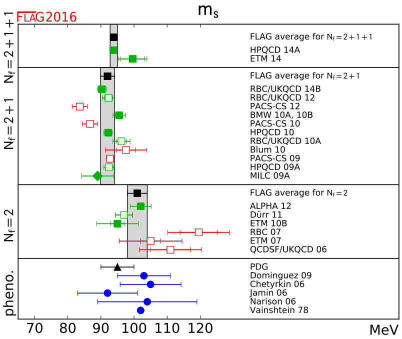
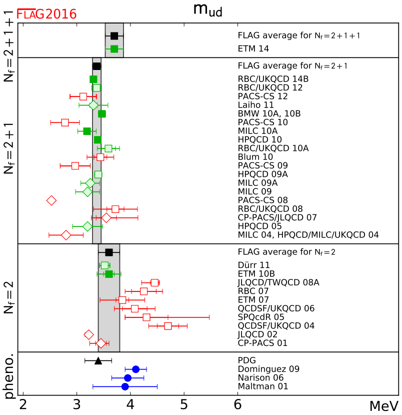
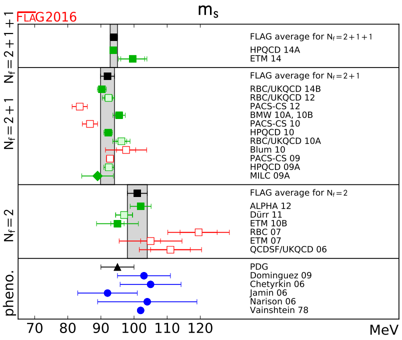




FLAG 2013 itpwiki.unibe.ch/flag/

Strategies for progress: gold plated quantities - a selection





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Summary

- Stable single-hadron states, below thresholds
- Including continuum extrapolation, realistic quark masses, renormalisation etc

Summary so far:

- The lattice provides a nonperturbative regulator for QCD and a framework for numerical simulation.
- In principle LQCD can go beyond models to provide a full description of states allowed by QCD - including exotic and excited states.
- Many challenges require not just bigger computers but better theory and algorthims.
- Lattice calculations of stable states demonstrate the power and precision that is possible with full control of error budgets.

Next: more recent developments in spectroscopy

- Look more deeply at some enabling ideas:
  - smearing and quark propagation
  - operator construction
  - resolution
- Recent results with these technologies

Smearing: beyond point propagators

The traditional idea - point propagators

Quark propagation from orgin to all sites on the lattice.

For better simulations of hadronic quantities look again at the building blocks: the quark propagators

Point propagator pros

- doesn't require vast computing resources

$$C(t, \mathbf{x}) = \langle Tr(\gamma_5 M_a^{-1}(x,0)^\dagger \gamma_5 \Gamma M_b^{-1}(x,0)\Gamma^\dagger)\rangle$$

Point propagator cons

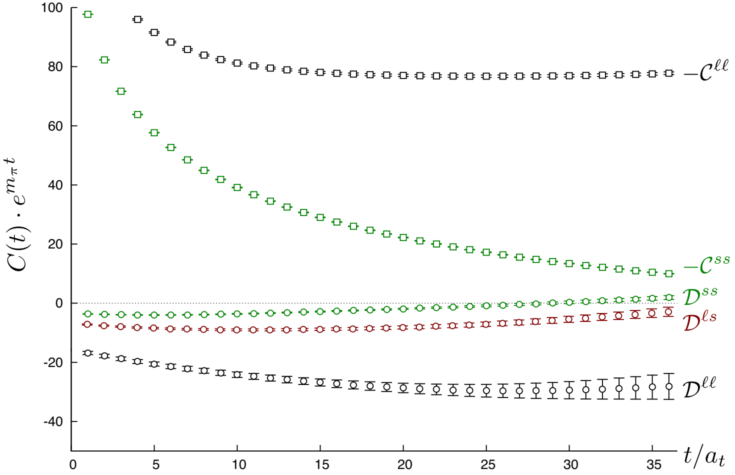
- restricts the accessible physics
  - flavour singlets and condenstates impossible: quark loops need props w sources everywhere in space
- restricts the interpolating basis used
  - a new inversion needed for every operator that is not restricted to a single lattice point
- entangles propagator calculation and operator construction
- throws away information encoded in configurations

Solutions?

- Improve the determination of point propagators to access the physics of interest: smearing
- Compute all elements of the quark propagator: all-to-all propagators. Problem It's expensive - needs an unrealistic number of inversions.
- Work around: Use stochastic estimators (with variance reduction). *[I won't talk about it here but see references for details]*
- *or* Rethink the problem: combine smearing and propagation i.e. **distillation** [More on this Friday.]

Distillation and disconnected spectroscopy

Isoscalar mesons



- Correlation functions for $\bar{\psi}\gamma_5\psi$ operator, with different flavour content (s, l).
- 16^3 lattice (about 2 fm). [arXiv:1102.4299]

Interpolating Operators

Constructing operators for the variational approach

A practical implementation of the variational method needs:

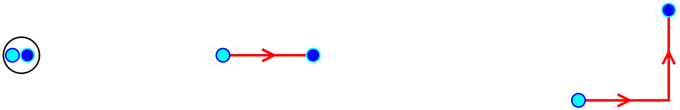
- a 'good' basis set that **resembles the states** of interest.
- **all elements** of this correlation matrix measured.

[see Blossier et.al. JHEP 0904 (2009) 094]

Extended Operators

- The simplest objects are colour-singlet local fermion bilinears: e.g. $\mathcal{O}_\pi = \bar{d}\gamma_5 u$
- Build more complicated operators with path-ordered using different offsets, connecting paths and spin contractions give different projections into lattice irreps.

Meson operators examples:

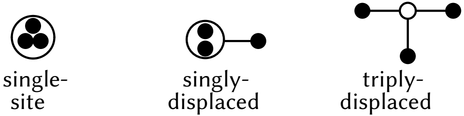


$\mathcal{O}_{\alpha\beta} = \sum_x \bar{\psi}_\alpha(x)\psi_\beta(x)$

$\mathcal{O}^{i}_{\alpha\beta} = \sum_x \bar{\psi}_\alpha(x)U_i(x)\psi_\beta(x+\hat{\imath})$

$\mathcal{O}^{ij}_{\alpha\beta} = \sum_x \bar{\psi}_\alpha(x)U_i(x)U_j(x+\hat{\imath})\psi_\beta(x+\hat{\imath}+\hat{\jmath})$

The same idea for baryons gives prototype extended operators:

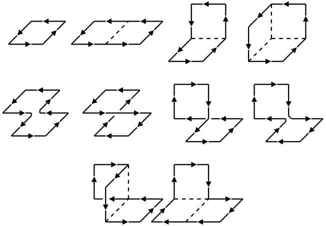


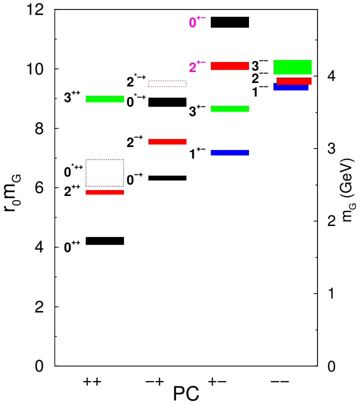
With thanks: 0810.1469

- We can make arbitrarily complicated operators in this way.
- An early success was glueball calculations

Glueballs

- QCD nonAbelian ⇒ allows bound states of glue
- Candidates observed experimentally: $f_0(1370)$, $f_0(1500)$, $f_0(2220)$
- Glueballs can be calculated in lattice QCD
- The interpolating fields are purely gluonic, built from Wilson loops





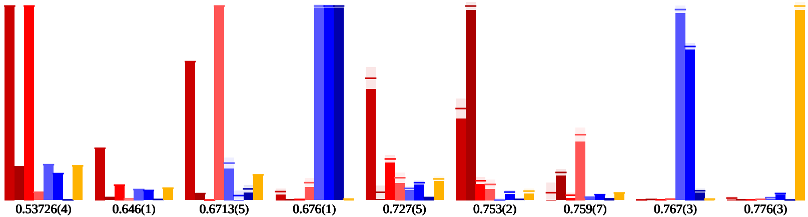
Morningstar and Peardon, 1999, Quenched

A strategy for operator construction (HadSpec Collab.)

- continuum operators of definite J^{PC} are subduced into the relevant irrep
- a subduced irrep carries a "memory" of continuum spin J from which it was subddduced - it **overlaps** predominantly with states of this J.

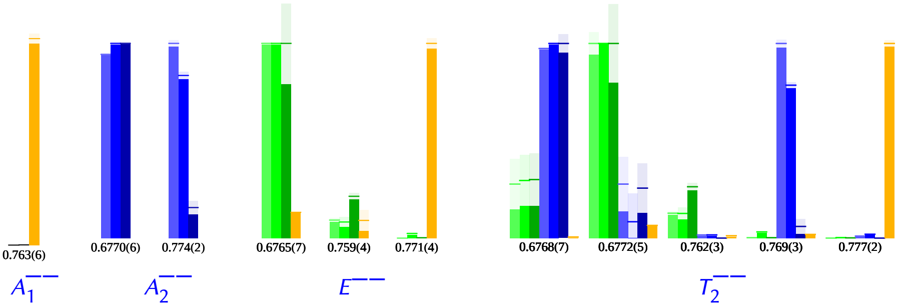
J	0	1	2	3	4
A_1	1	0	0	0	1
A_2	0	0	0	1	0
E	0	0	1	0	1
T_1	0	1	0	1	1
T_2	0	0	1	1	1

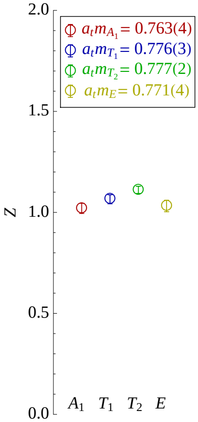
- Using $Z = \langle 0|\Phi|k\rangle$, helps to identify continuum spins
- For high spins, can look for agreement between irreps
- Data below for T_1^{--} irrep, colour-coding is **Spin 1**, **Spin 3** and **Spin 4**.



. . . the rest of the spin-4 state

- All polarisations of the spin-4 state are seen
- Spin labelling: **Spin 2**, **Spin 3** and **Spin 4**.





Improving resolution - the anisotropic lattice

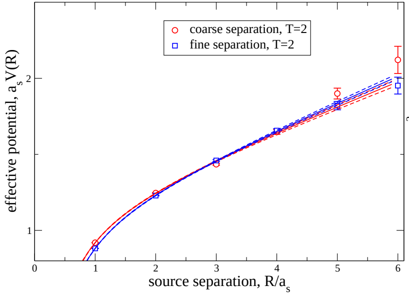
- If we can build a good basis of operators, we can extract energies of low-lying states from the correlator at short distances.
- The lattice correlator can only be sampled at discrete values of t and signal falls quickly for a massive state, while the statistical noise does not. Reducing the lattice spacing is extremely computationally expensive
- Mitigate this cost by reducing just the temporal lattice spacing, keeping the spatial mesh coarser; the anisotropic lattice.
- Unfortunately this reduces the symmetries of the theory from the hypercubic to the cubic point group. The dimension four operators on the lattice now split;

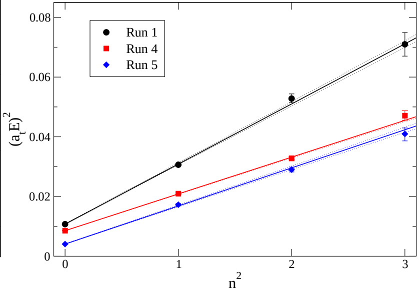
$$\begin{aligned} \mathrm{Tr}\, F_{\mu\nu}F_{\mu\nu} &\rightarrow \left\{\mathrm{Tr}\, F_{ij}F_{ij}, \mathrm{Tr}\, F_{i0}F_{i0}\right\} \\ \bar{\psi}\gamma_\mu D_\mu \psi &\rightarrow \left\{\bar{\psi}\gamma_i D_i \psi, \bar{\psi}\gamma_0 D_0 \psi\right\} \end{aligned}$$

- On $3 \oplus 1$ anisotropic lattices, spatial symmetries unchanged.

QCD and the anisotropic lattice

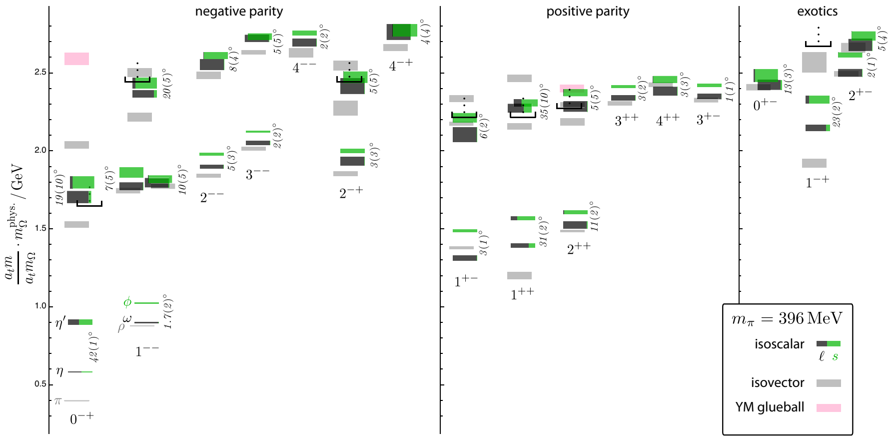
- The space-time symmetry breaking in QCD introduces extra bare parameters ($\xi = a_s/a_t$) in the lagrangian, that must be tuned to restore Euclidean rotational invariance in the continuum limit.
- For QCD, both the quarks and gluons must "feel" the same anisotropy; this requires tuning *a priori*.
- Two physical conditions are satisfied simultaneously, derived from the "sideways" potential and the pion dispersion relation.





Results:
single hadron states

Light isoscalar meson spectrum



from HSC 2012

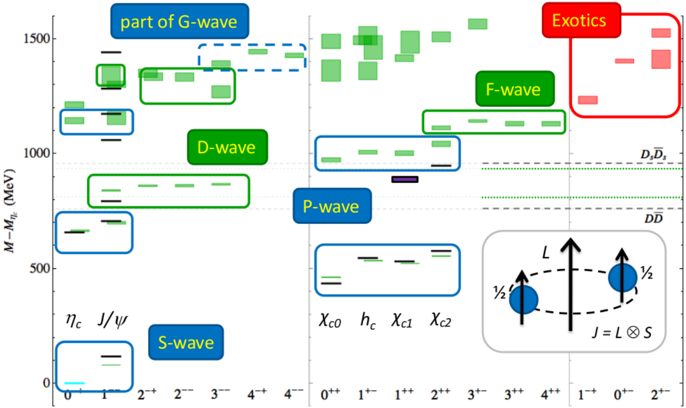
Charmonium: excited and exotic states

Precision calculation of high spin ($J \geq 2$) and exotic states is relatively new.

Caveat Emptor

- Only single-hadron operators
- Physics of multi-hadron states appears to need relevant operators
- No continuum extrapolation
- Relatively heavy pions ← already changing

Charmonium



from HSC 2012

Interim summary

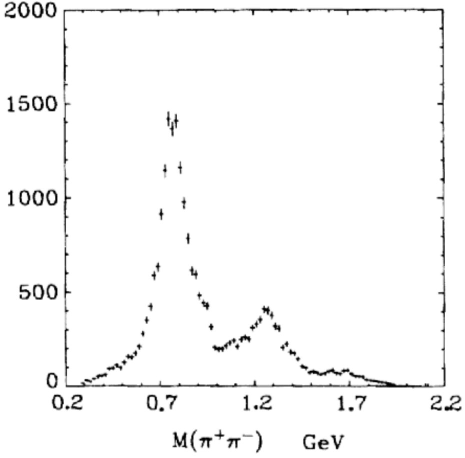
- New ideas are enabling rapid progress.
- Lots of precision and pioneering calculations of hadronic quantities.
- Next - multi-hadron, many-body systems. A rapidly moving field!

Resonances and scattering states

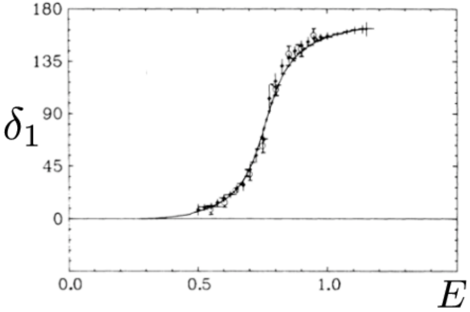
- We have assumed that all states in the spectrum are stable
- Many (the majority) are not.
- A resonance is a state that forms e.g. when colliding two particles and then decays quickly to scattering states.
- They respect conservation laws: if isospin of the colliding particles is 3/2, resonance must have isospin 3/2 (Δ resonance)
- Usually indicated by a sharp peak in a cross section as a function of c.o.m. energy of the collision.

How can lattice QCD distinguish resonances and scattering states?

Excited hadrons are resonances

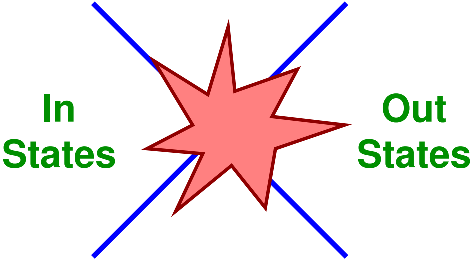


Resonant phase shift



Resonances are pole singularites in complex $s = E^2$

Maiani-Testa no-go theorem

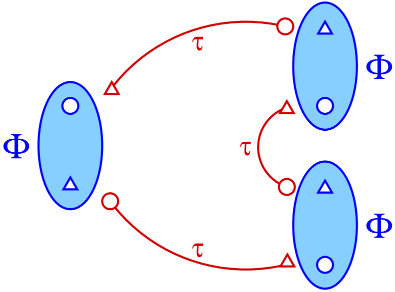


- Monte-carlo simulations via importance sampling rely on a path integral with positive definite probability measure: Euclidean space.
- Maiani-Testa: scattering matrix (S-matrix) elements cannot be extracted from infinite-volume Euclidean-space correlation functions (except at threshold).

Michael 1989 and Maiani, Testa (1990)

Maiani-Testa (2)

- Can understand this since:
  - Minkowski space: S-matrix elements complex functions above kinematic thresholds.
  - Euclidean space: S-matrix elements are real for all kinematics - phase information lost.
- Lattice simulations with dynamical fermions admit strong decays e.g. for light-enough up and down dynamical quarks $\rho \to \pi\pi$



Maiani-Testa (3)

- Can the lattice get around the no-go theorem to extract the masses and widths of such unstable particles?
- Yes - **use** the finite volume.
- Computations done in a periodic box
  - momenta quantised
  - discrete energy spectrum of stationary states → single hadron, 2 hadron ...
  - scattering phase shifts → resonance masses, widths deduced from finite-box spectrum
    - B. DeWitt, PR 103, (1956) - sphere
    - M. Lüscher, NPB (1991) - cube
  - Two-particle states and resonances identified by examining the behaviour of energies in finite volume

For elastic two-body resonances (Lüscher): $M_1 M_2 \rightarrow R \rightarrow M_1 M_2$

⟶ Volume dependence of energy spectrum
⟶ Phase shift in infinite volume
⟶ Mass and width of resonance - parameterising the phase shift e.g. with Breit Wigner.

Scattering in a Euclidean Field Theory

Recall, lose direct access to scattering in (Euclidean) lattice calculations.

- The large Euclidean time limit dominated by threshold or off-shell states. Maiani-Testa, PLB (1990).
- Analytic continuation of numerical (euclidean) correlators is an ill-posed problem.

Lüscher found a way to extract $\pi\pi \to \pi\pi$ scattering information in the elastic region from LQCD. [NPB354, 531-578 (1991)]

- Use the finite volume as a tool
- Related lattice energy levels in a finite volume to a decomposition of the scattering amplitude in partial waves in infinite volume

$$\det\left[\cot\delta(E_n^*) + \cot\phi(E_n, \vec{P}, L)\right] = 0$$

  and $\cot\phi$ a known function (containing a generalised zeta function).
- Also known from perturbative NR-QM. Huang-Yang PRD105 (1957)
- To use this idea many technical improvements were needed. Eg need disconnected diagrams and energy levels at extraordinary precision. This is why it has taken a while

The method has been generalised for

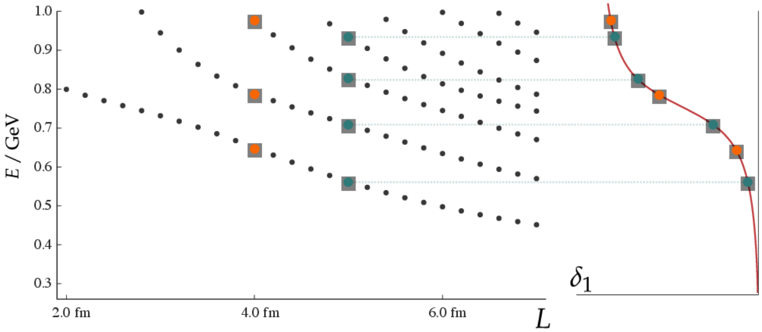
- **moving frames; non-identical particles; multiple two-particle channels, particles with spin**

Rummukainen and Gottlieb, Nucl. Phys. B450, 397 (1995)
Beane, Bedaque, Parreno, and Savage, Nucl. Phys. A747, 55 (2005)
Kim, Sachrajda, and Sharpe, Nucl. Phys. B727, 218 (2005)
Christ, Kim, Yamazaki, Phys. Rev. D72, 114506 (2005)
Bernard, Lage, Meissner, and Rusetsky, JHEP, 1101, 019 (2011)
Hansen and Sharpe, Phys.Rev. D86 (2012) 016007
Briceño and Davoudi, Phys.Rev. D88 (2013) 094507
Li and Liu, Phys. Rev. D87, 014502 (2013)
Briceño, Phys. Rev. D 89, 074507 (2014)

$\pi\pi$ in P-wave: $I^G J^{PC} = 1^+(1^{--})$

A relatively straightforward example

- consider the *rho* on a lattices of different volumes
- at each volume extract the spectrum and use Luscher to deduce phase shift

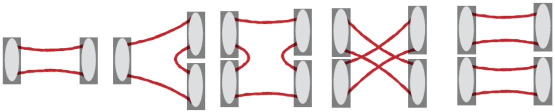


- the more distinct spectrum points the better the phase shift picture

Mapping out the phase shift

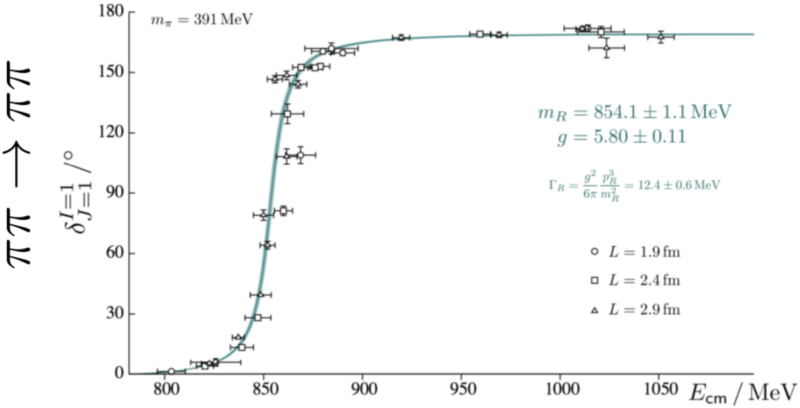
What sort of operators are needed? A good operator basis reflects the relevant physics.

- "single hadron": $\bar{\psi}\Gamma\psi \sim \bar{\psi}\Gamma\overleftrightarrow{D}\ldots\overleftrightarrow{D}\psi$ with the correct quantum numbers
- "multi-hadron": $\mathcal{O}_{\pi\pi}(|\vec{p}|) = \sum_{\hat{p}} c(\hat{p})\pi(\vec{p})\pi(-\vec{p})$ where each π operator is a "variationally optimised" object, $\pi = \sum_i v_i(\bar{u}\Gamma_i d)$ for a range of $|p|$
- form a correlator matrix with these operators and diagonalise in a "variational" way



$\pi\pi$ in P-wave I=1

A Breit-Wigner energy-dependent description

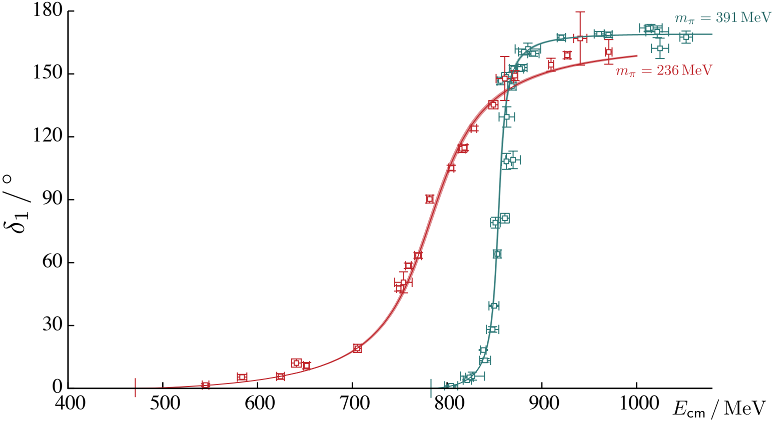


$$m_\pi = 391 \text{ MeV}$$

from Dudek, Edwards, Thomas in *Phys.Rev.* D87 (2013) 034505

$\pi\pi$ in P-wave, I=1

coupled $\pi\pi$, $K\bar{K}$ scattering in P-wave, PRD 92 (2015) 094592



- Includes **coupled channel** $\pi\pi$, $K\bar{K}$ at $m_\pi = 236$MeV.
- $m_R = 790(2)$ MeV; $g_R = 5.688(70)(26)$
- Reducing m_π moves mass and width in the right direction.

Summary & Outlook

- Reviewed lattice framework and results showing convergence of results from different methods.
- Understood how to extract information from correlators and some details of fitting.
- Discussed the practicalities of lattice calculations and consequences.
- New ideas are enabling rapid progress.
- Lots of precision and pioneering calculations of hadronic and nuclear quantities.
- Next - exotic states, what can the lattice say?!

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Thanks for listening!