Practical Numerical Solutions

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Abstract of the course

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1 adaptive error algorithm

$$\bar{x}(t;h) = x(t) + h^p e_p(t) + \dots$$
$$\bar{x}(t;\frac{h}{2}) = x(t) + \left(\frac{h}{2}\right)^p e_p(t)$$
$$\bar{x}(t;h) - \bar{x}(t;\frac{h}{2} = e_p(t)(h^p - \frac{h}{2})$$
$$= e_p(t)h^p(1 - \frac{1}{2^p})$$
$$= e_p(t)h^p \left(\frac{2^p - 1}{2^p}\right)$$

$$\varepsilon(t;\frac{h}{2}) = e_p(t) = \frac{\bar{x}(t;h) - \bar{x}(t;\frac{h}{2})}{2^p - 1}$$

2 Leap frog intergrator

$$\dot{\dot{x}} = -x$$

$$x^{(1)} = \dot{x}$$
$$\dot{x}^{(1)} = -x$$
$$\dot{x}^{(0)} = x^{(1)}$$
$$\dot{x}^{(1)} = -x^{(0)}$$
$$x_{k+1}^{(0)} = x_k^{(0)} + hx_k^{(1)}$$
$$x_{k+1}^{(1)} = x_k^{(1)} + hx_k^{(1)}$$

$$\begin{pmatrix} x_{k+1}^{(0)} \\ x_{k+1}^{(1)} \end{pmatrix} = \begin{pmatrix} 1 & h \\ -h & 1 \end{pmatrix} \begin{pmatrix} x_k^{(0)} \\ x_k^{(1)} \end{pmatrix}$$

$$M = SDS^{-1}$$
$$\begin{pmatrix} 1 & h \\ -h & 1 \end{pmatrix}$$

Tr(M)=2
det(M)=$1 + h^2$

$$\lambda = \{1 + ih, 1 - ih\}$$
$$S = \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

$$M^k = \frac{r^k}{2} S \Delta S^{-1}$$
$$\Delta = \begin{pmatrix} e^{i\delta k} & o \\ 0 & e^{-i\delta k} \end{pmatrix}$$
$$\delta = \tan^{-1} h$$
$$r = \sqrt{1 + h^2}$$

Euler Integrator always unstable for all values of h.

3 Symplectic Integrator

Leap frog. see picture in Ginna's copy!! can have integrators that dont blow up with this tactic, thus will be able to do our homework on planets around the sun.