Dimensional Regularisation

Consider ϕ^4-theory with Lagrangian

$$\mathcal{L} = \partial_\mu \phi(x) \partial^\mu \phi(x) - \frac{1}{2} m^2 \phi^2(x) - \frac{1}{4!} \lambda \phi^4(x) \tag{1}$$

where $\phi(x)$ is a real-valued scalar field of mass m, and λ is a dimensionless coupling constant.

(a) Draw all Feynman diagrams for the Møller scattering process

$$\phi + \phi \to \phi + \phi$$

to 1-loop order and show that the invatiant scattering amplitude is of the form

$$i\mathcal{M}\left(\phi(p) + \phi(q) \to \phi(p') + \phi(q')\right) = -i\lambda - i\lambda^2 [iV(s) + iV(t) + iV(u)] \tag{2}$$

where s, t and u are the usual Mandelstam variables.

(b) Write down an expression for the t-channel scattering amplitude $V(t)$ to 1-loop order in terms of a single Feynman parameter[1] and an undetermined loop-momentum.

(c) By performing a Wick rotation[2] use Dimensional Regularisation[3] to integrate out the undetermined loop-momentum to reduce the expression for $V(t)$ to an integral over a single Feynman parameter only.

[1] Recall that by Feynman

$$\frac{1}{A_1} \frac{1}{A_2} \cdots \frac{1}{A_n} = \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \cdots \int_0^1 \mathrm{d}x_n \, \delta\left(-1 + \sum_{i=1}^n x_1\right) \frac{(n-1)!}{[x_1 A_1 + x_2 A_2 + \cdots + x_n A_n]^n}$$

[2] Recall that by Wick letting

$$\ell^\mu \to \ell^\mu_\mathcal{E} = \left(-i\ell^0, \vec{\ell}\right)$$

gives

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}^4 \ell}{(2\pi)^4} \frac{1}{[\ell_\mu \ell^\mu - \Delta]^m} = \frac{i}{(-1)^m} \frac{1}{(2\pi)^4} \int_{-\infty}^{\infty} \mathrm{d}^4 \ell_\mathcal{E} \, \frac{1}{[(\ell_\mathcal{E})_\mu \ell^\mu_\mathcal{E} + \Delta]^m}$$

[3] Recall that the Euler β-function is given by

$$\begin{aligned} \beta(p, q) &= \int_0^1 \mathrm{d}x \, x^{p-1} (1-x)^{q-1} \\ &= \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \end{aligned}$$

where

$$\Gamma(x) = \int_0^\infty \mathrm{d}x \, x^{t-1} \exp(-x)$$

is the usual Gauss γ-function.