## Pseudoscalar-Yukawa Theory

Consider Pseudoscalar-Yukawa Theory with Lagrangian

$$\mathcal{L} = \partial_\mu \phi(x) \partial^\mu \phi(x) - \frac{1}{2}\mu^2 \phi^2(x) + \bar{\psi}(x) \left[ i\gamma^\mu \partial_\mu - m \right] \psi(x) - g\bar{\psi}(x)\gamma^5 \psi(x)\phi(x) \quad (1)$$

where $\phi(x)$ is a real-valued scalar field of mass $\mu$, $\psi(x)$ is a spinor field of mass $m$, and $g$ is a dimensionless coupling constant.

(a) Write down the Feynman rules for Pseudoscalar-Yukawa Theory.

(b) Draw all Feynman diagrams for the Møller scattering process

$$\psi + \psi \to \psi + \psi$$

to tree-level order[1] and calculate the unpolarised spin-averaged sum-over-spins[2] invariant scattering amplitude[3].

(c) Draw all Feynman diagrams for the Bhabha scattering process

$$\psi + \bar{\psi} \to \psi + \bar{\psi}$$

to tree-level order and calculate the unpolarised spin-averaged sum-over-spins invariant scattering amplitude.

(d) Calculate the differential scattering cross-section[4] for both scattering processes.

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[1] Recall that the usual Mandelstam variables $s$, $t$ and $u$ are given by

$$s = (p+q)_\mu (p+q)^\mu \quad \text{and} \quad t = (p-p')_\mu (p-p')^\mu \quad \text{while} \quad u = (p-q')_\mu (p-q')^\mu$$

where $p$ and $q$ are the initial-state momenta while $p'$ and $q'$ are the final-state momenta.

[2] Recall that

$$\sum_s u(p,s) \otimes \bar{u}(p,s) = \gamma^\mu p_\mu + I_4 m \quad \text{and} \quad \sum_r v(q,r) \otimes \bar{v}(q,r) = \gamma^\mu q_\mu - I_4 m$$

[3] Recall that we may always write an amplitude $A$ as

$$A = (2\pi)^4 \delta^{(4)}\Big(p + q - \sum_f p_f\Big) i\mathcal{M}$$

where $p$ and $q$ are the initial-state momenta while $p_f$ are the final-state momenta.

[4] Recall that in the centre-of-mass frame

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{CM}} = \frac{1}{64\pi^2 s} |\mathcal{M}|^2$$