Quantum Electrodynamics

Bhabha Scattering

Consider Quantum Electrodynamics with Lagrangian

$$\mathcal{L} = \bar{\psi}(x)\left[i\gamma^\mu \mathcal{D}_\mu - m\right]\psi(x) - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} \tag{1}$$

where $\psi(x)$ is a spinor field of mass m,

$$\mathcal{D}_\mu = \partial_\mu - ieA_\mu(x)$$

with $A_\mu(x)$ a massless 1-form gauge field, e a dimensionless coupling constant, while

$$\mathcal{F}_{\mu\nu} = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$$

is the field-strength tensor.

(a) Draw all Feynman diagrams for the Bhabha scattering muon-antimuon[1] creation process

$$e^+ + e^- \to \mu^+ + \mu^-$$

to tree-level order and calculate the polarised[2] sum-over-spins[3] invariant scattering amplitude[4].

(b) Calculate the differential scattering cross-section[5] for all ingoing and outgoing polarisations.

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[1] Recall that

$$m_e \simeq \frac{1}{200} m_\mu$$

where m_e is the electron mass and m_μ is the muon mass.

[2] Recall that the projection operators P_+ and P_- give states of definite helicity, where

$$P_+ = \frac{1}{2}(I_4 + \gamma^5) \quad \text{and} \quad P_- = \frac{1}{2}(I_4 - \gamma^5)$$

and that P_+ and P_- and idempotent and orthogonal such that

$$P_+^2 = P_+ \quad \text{and} \quad P_-^2 = P_-$$

and

$$P_+ P_- = 0 \quad \text{and} \quad P_- P_+ = 0$$

[3] Recall that

$$\sum_s u(p,s) \otimes \bar{u}(p,s) = \gamma^\mu p_\mu + I_4 m \quad \text{and} \quad \sum_r v(q,r) \otimes \bar{v}(q,r) = \gamma^\mu q_\mu - I_4 m$$

[4] Recall that we may always write an amplitude A as

$$A = (2\pi)^4 \delta^{(4)}\Big(p + q - \sum_f p_f\Big) i\mathcal{M}$$

where p and q are the initial-state momenta while p_f are the final-state momenta.

[5] Recall that in the centre-of-mass frame

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{CM}} = \frac{1}{64\pi^2 s}|\mathcal{M}|^2$$