04/02/2015 MA 1214 Group Theory - Homework 3

1\. (i) - We wish to find ab and a^{-1}

~ Well,

$$ab = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 3 & 2 & 5 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 1 & 2 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 1 & 4 & 2 \end{pmatrix}$$

while

$$a^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 3 & 2 & 5 \end{pmatrix}$$

$$= a$$

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(ii) - We wish to solve

$$ax = b$$

~ Thus

$$x = a^{-1}b$$
$$= ab$$
$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 1 & 4 & 2 \end{pmatrix}$$

(iii) - We wish to write b as a product of transpositions

~ Well,

$$b = (13)(25)(54)$$

and this is odd

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2. (I) - We wish to check if S_3 is cyclic

~ But $(1\,2) \in S_3$ and $(1\,3) \in S_3$ with
$$(1\,3) \neq (1\,2)^n$$
for any $n \in \mathbb{N}$

~ Thus S_3 is not cyclic

(II) - We wish to check if $n\mathbb{Z}$ is cyclic

~ Well,
$$n\mathbb{Z} = \langle n \rangle$$

(III) - We wish to check if $\mathbb{Z}$ is cyclic

~ Well, $\mathbb{Z}$ is an infinite cyclic group with
$$\mathbb{Z} = \langle 1 \rangle$$

(IV) - We wish to check if $\mathbb{R}$ is cyclic

~ But given $\varepsilon \in \mathbb{R}$ with
$$\varepsilon > 0$$
we have $\frac{1}{2}\varepsilon \in \mathbb{R}$, and so there is no ε which generates $\mathbb{R}$

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2. (v) - We wish to check if $G \subset GL(2, \mathbb{Q})$ is cyclic where

$$g = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$$

for all $g \in G$

- But given $r \in \mathbb{Q}$ there exists $\frac{1}{2}r \in \mathbb{Q}$, and so there is no r which generates $\mathbb{Q}$

3. (i) - We wish to write
$$a=\begin{pmatrix}1&2&3&4&5\\3&5&1&2&4\end{pmatrix}$$
as a product of disjoint cycles

~ Well,
$$a=(13)(254)$$
and this is odd

(ii) - We wish to write
$$b=\begin{pmatrix}1&2&3&4&5&6\\3&5&1&6&2&4\end{pmatrix}$$
as a product of disjoint cycles

~ Well,
$$b=(13)(25)(46)$$
and this is odd

(iii) - We wish to write
$$c=(12)(234)(34567)$$
as a product of disjoint cycles

~ Then
$$c=(123)(4567)$$
and this is odd