21/01/2015 MA 1214 Group Theory - Homework 1



1. (i) - We wish to find how many maps, how many injective maps, how many surjective maps, and how many bijective maps

$$f: \{1\} \to \{1, 2\}$$

exist

~ Well, we have 2 possible maps

~ Then we have 2 injective maps

~ But we have 0 surjective maps

~ Thus we have 0 bijective maps





(ii) - We wish to find how many maps, how many injective maps, how many surjective maps, and how many bijective maps

$$f: \{1, 2\} \to \{1, 2\}$$

~ Well, we have 4 possible maps

~ Then we have 2 injective maps

~ Moreover we have 2 surjective maps

~ Thus we have 2 bijective maps

(iii) - We wish to find how many maps, how many injective maps, how many surjective maps, and how many bijective maps exists $f : \{1, 2\} \to \{1, 2, 3\}$

- Well, we have 9 possible maps
- Then we have 6 injective maps
- But we have 0 surjective maps
- Thus we have 0 bijective maps

16/03/2015 MA 1214 Group Theory - Homework 1



2. (i) - We wish to find the inverse map for

$$f : \mathbb{R} \to \mathbb{R}$$
$$x \mapsto f(x) = -4x$$

~ Well,

$$f^{-1} : \mathbb{R} \to \mathbb{R}$$
$$y \mapsto f^{-1}(y) = -\tfrac{1}{4}y$$

(ii) - We wish to find the inverse map for

$$f : \mathbb{R} \to \mathbb{R}$$
$$x \mapsto f(x) = 2x + 2$$

~ Thus

$$f^{-1} : \mathbb{R} \to \mathbb{R}$$
$$y \mapsto f^{-1}(y) = \tfrac{1}{2}y - 1$$

(iii) - We wish to find the inverse map for

$$f : \mathbb{R} \to \mathbb{R}^+$$
$$x \mapsto f(x) = \exp(x-1)$$

~ Then

$$f^{-1} : \mathbb{R}^+ \to \mathbb{R}$$
$$y \mapsto f^{-1}(y) = \log(y) + 1$$

3. (i) - We wish to show that
$$f(A \cup B) = f(A) \cup f(B)$$
where
$$f: S \to T$$
is a map with $A \subseteq S$ and $B \subseteq S$ subsets

~ Well, let $y \in f(A \cup B)$
Then there exists some $x \in A \cup B$ such that
$$y = f(x)$$

~ But here $x \in A$ or $x \in B$, and so $f(x) \in f(A)$ or $f(x) \in f(B)$

~ Thus $f(x) \in f(A) \cup f(B)$, or rather $y \in f(A) \cup f(B)$

~ However this is true for all $y \in f(A \cup B)$, whence
$$f(A \cup B) \subseteq f(A) \cup f(B)$$

~ On the other hand, let $y \in f(A) \cup f(B)$
Then either there exists some $x \in A$ such that
$$y = f(x)$$
or there exists some $x \in B$ such that
$$y = f(x)$$

~ That is, there exists some $x \in A \cup B$ such that
$$y = f(x)$$

~ But here $f(x) \in f(A \cup B)$, or rather $y \in f(A \cup B)$, and this is true for all $y \in f(A) \cup f(B)$, whence
$$f(A) \cup f(B) \subseteq f(A \cup B)$$

21/01/2015 MA 1214 Group Theory - Homework 1



3. ~ But if

$$f(A \cup B) \subseteq f(A) \cup f(B) \quad \text{and} \quad f(A) \cup f(B) \subseteq f(A \cup B)$$

then we must have

$$f(A \cup B) = f(A) \cup f(B)$$

as desired

$\blacksquare$

(ii) - We wish to show that
$$f(A \cap B) \subseteq f(A) \cap f(B)$$
where
$$f: S \to T$$
is a map with $A \subseteq S$ and $B \subseteq S$ subsets

- Well, let $y \in f(A \cap B)$
Then there exists $x \in A \cap B$ such that
$$y = f(x)$$

- That is, there exists both $x_a \in A$ such that
$$y = f(x_a)$$
and there exists $x_b \in B$ such that
$$y = f(x_b)$$
with
$$x_a = x_b$$

- But $f(x_a) \in f(A)$ and $f(x_b) \in f(B)$ with
$$f(x_a) = f(x_b)$$
and so $f(x_a) \in f(A) \cap f(B)$, or rather,
$y \in f(A) \cap f(B)$

- However this is true for all $y \in f(A \cap B)$, and so we must have that
$$f(A \cap B) \subseteq f(A) \cap f(B)$$
as desired ■

21/03/2015 MA 1214 Group Theory - Homework 1



4. (i) - We wish to check if

$$* : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$$
$$(m,n) \mapsto m*n = mn - 1$$

is commutative and associative

~ Well,

$$n*m = nm - 1$$
$$= mn - 1$$
$$= m*n$$

and so $*$ is commutative

~ However,

$$(m*n)*p = (mn-1)*p$$
$$= (mn-1)p - 1$$
$$\neq m(np-1) - 1 = m*(n*p)$$

and so $*$ is not associative





(ii) - We wish to check if the binary operation

$$* : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$$
$$(m,n) \mapsto m*n = \tfrac{1}{2}(m+n)$$

is commutative and associative

~ Well,

$$n*m = \tfrac{1}{2}(n+m)$$
$$= \tfrac{1}{2}(m+n)$$
$$= m*n$$

and so $*$ is commutative

~ However,



$$(m*n)*p = [\tfrac{1}{2}(m+n)]*p$$

or

$$(m*n)*p = \tfrac{1}{2}[\tfrac{1}{2}(m+n)+p]$$
$$\neq \tfrac{1}{2}[m+\tfrac{1}{2}(n+p)] = m*(n*p)$$

and so $*$ is not associative

(iii) - We wish to check if the binary operation

$$*: \mathbb{N}\times\mathbb{N} \to \mathbb{N}$$
$$(m,n) \mapsto m*n = 5$$

is commutative and associative

~ Well,

$$n*m = 5$$
$$= m*n$$

and so $*$ is commutative

~ Furthermore,

$$(m*n)*p = 5*p$$
$$= 5$$
$$= m*(n*p)$$

and so $*$ is associative