March 23, 2015 Lecturer Dmitri Zaitsev Hilary Term 2015

Course 1214 - Introduction to group theory 2015

Sheet 9

Due: at the end of the lecture

Exercise 1

(i) For any groups G_1 and G_2 prove that $G_1 \times \{e\}$ is a normal subgroup in $G_1 \times G_2$.

(ii) Show that the quotient group $(G_1 \times G_2)/(G_1 \times \{e\})$ is isomorphic to G_2.

(ii) Prove that the intersection of all normal subgroups in a group is again a normal subgroup.

Exercise 2

Consider the action of $G := \mathbb{Z}$ on $S := \mathbb{Z}_2 \times \mathbb{Z}_4$ given by

$$z \cdot ([a], [b]) := ([z + a], [z + b]).$$

(i) Show that this indeed is a well-defined group action.

(iii) What are the orbits $G \cdot (a, b)$ and what are the stabilizers $G_{(a,b)}$ for $(a, b) \in S$?

Exercise 3

Determine the order of each of the following quotient groups:

(i) $\mathbb{Z}_8/\langle[2]\rangle$

(ii) $\mathbb{Z}_8/\langle[3]\rangle$.

Are these groups cyclic?