January 21, 2013 Lecturer Dmitri Zaitsev Hilary Term 2015

Course 1214 - Introduction to group theory 2015

S h e e t 1

Due: at the end of the lecture

Exercise 1

How many maps, injective maps, surjective maps and bijective maps f from A to B exist for

(i) $A = \{1\}$, $B = \{1, 2\}$;
(ii) $A = \{1, 2\}$, $B = \{1, 2\}$;
(iii) $A = \{1, 2\}$, $B = \{1, 2, 3\}$.

Exercise 2

Find the inverse map f^{-1} and specify its source and target for

(i) $f(x) = -4x$;
(ii) $f(x) = 2x + 2$;
(iii) $f(x) = e^{x-1}$.

Exercise 3

Let $f\colon S \to T$ be a map and $A, B \subset S$ be two subsets.

(i) Show that $f(A \cup B) = f(A) \cup f(B)$;
(ii) Show that $f(A \cap B) \subset f(A) \cap f(B)$ and illustrate by example that "$\subset$" cannot be replaced by "$=$".

Exercise 4

Which binary operations $*$ on the natural numbers $\mathbb{N}$ are commutative and which are associative:

(i) $m * n = mn - 1$;
(ii) $m * n = \frac{m+n}{2}$;
(ii) $m * n = 5$.