The Cosmological Chameleon

A Scalar-Tensor Theory of Gravity & Dark Energy

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- Einstein's Theory of General Relativity[1, 2, 3] - Gravity
- The Big Bang Theory[4, 5] - A Universe which Expands
- Type IA Supernovae - Giant Star Explosions!
- The Expansion is Accelerating![6, 7]

Possible Explinations

The Astrophysicist Approach

- Step 1: Measure Accelerating Cosmic Expansion
- Step 2: ???
- Step 3: Dark Energy!

Possible Explinations

The Theoretical Physicist Approach

- Step 1: Accelerating Cosmic Expansion
- Step 2: Maths!
- Step 3: Predict Dark Energy

Possible Explinations
The Theoretical Physicist Approach

- Lots and lots of maths!

$$S = \int d^4x \sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}\mathcal{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right) - \int d^4x\, \mathcal{L}_m(\psi_m^{(i)}, g_{\mu\nu}^{(i)}) \quad (1)$$

where

$$M_{\rm Pl} = \frac{1}{\sqrt{8\pi G}} \simeq 10^{18}\,\mathrm{GeV} \quad (2)$$

is the *Reduced Planck Mass* in natural units such that $h = c = 1$ with $\mathcal{L}_m$ the Lagrangian for the matter fields $\psi_m^{(i)}$.

We assume a conformal coupling of the chameleon field to the matter fields $\psi_m^{(i)}$ of species i. That is, the chameleon field gives rise to a fifth force. Then

$$g_{\mu\nu}^{(i)} = \exp\left(\frac{2\beta_i\phi}{M_{\rm Pl}}\right) \quad (3)$$

where $g_{\mu\nu}$ is the Einstein frame metric tensor, with β_i being some constant. This gives a Klein-Gordon field equation

$$\nabla^2\phi = V_{,\phi}(\phi) - \sum_i \frac{\beta_i}{M_{\rm Pl}} \exp\left(\frac{4\beta_i\phi}{M_{\rm Pl}}\right) g_{\mu\nu}(i) T_{\mu\nu}^{(i)} \quad (4)$$

where

$$T^{(i)} = \frac{2}{\quad} \frac{\delta\mathcal{L}_m}{\quad}$$

which is a combination of the scalar field potential and a term proportional to the matter energy-density such that

$$V_{\rm Eff} = V_{\rm Eff}(\rho_i)$$

Now, if either the potential $V(\phi)$ is monotonically decreasing with ϕ and $\beta_i > 0$, or, if $V(\phi)$ is monotonically increasing with ϕ and $\beta_i < 0$, then we see that the effective potential $V_{\rm Eff}(\phi)$ has a minimum for some value $\phi_{\rm min}$ of the chameleon field such that

$$V_{{\rm Eff},\phi}(\phi_{\rm min}) = V_{,\phi}(\phi_{\rm min}) + \sum_i \frac{\beta_i}{M_{\rm Pl}}\rho_i \exp\left(\frac{\beta_i\phi_{\rm min}}{M_{\rm Pl}}\right) = 0 \quad (7)$$

For small fluctuations about the minimum of the effective potential we get a mass term

$$m_\phi^2 = V_{{\rm Eff},\phi\phi}(\phi_{\rm min}) = V_{,\phi\phi}(\phi_{\rm min}) + \sum_i \frac{\beta_i^2}{M_{\rm Pl}^2}\rho_i \exp\left(\frac{\beta_i\phi_{\rm min}}{M_{\rm Pl}}\right) \quad (8)$$

We let $V(\phi)$ be a runaway Ratra-Peebles potential such that

$$V(\phi) = M^4 f\left(\frac{\phi}{M}\right) \quad (9)$$

say, where M is the scale of the interaction, and f is some function which gives a quintessence

- The Chameleon Particle[8, 9, 11, 12] - a Slim Scalar Field $\phi(x)$
- Effective Potential Energy Function V_{Eff} which is a combination of the Scalar Field Potential and a term proportional to the Matter Energy-density ρ

$$V_{\text{Eff}} = V_{\text{Eff}}(\rho)$$

- Field changes in accordance with its background - Chameleon![1]

- Astrophysicists Search

- Experiments set an Upper Bound

$$M \lesssim 10^{-3}\text{eV}$$

The Chameleon Model
It Works!

- By Maths → Particles go the the Minimum of the Potential Energy

- We find that both Locally and on Cosmological Scales

$$M^4 \simeq 10^{-12}\text{eV}$$

- Same order as the Vacuum Energy Density!

Conclusions
The Chameleon Model

- Step 1: Type IA Supernovae - Accelerating Cosmic Expansion
- Step 2: Chameleon Particle - Mass scales with Local Matter Density
- Step 3: Vacuum Energy Density scale $\simeq 10^{-12}$eV
- Future Work - Hunt for the Chameleon
  Get to work Astros!

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Any Questions???

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