RC Filter Networks

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8th April 2010

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1 Abstract

In this experiment the response of both *High Pass* and *Low Pass RC Filter Networks* to both sinusoidal signals and square wave signals were investigated by examining how the output signal changes as the frequency is varied. The *Half Power Point* frequencies for each of the filter networks were measured, and were found to be 1100 ± 50 and 940 ± 5 respectively, corresponding to values of $\log_{10} f_{high} = 3.04 \pm 0.5$ and $\log_{10} f_{low} = 2.973 \pm 0.005$. Finally, the *Time Constant* for the low pass filter network was measured and it was found to be $\tau = 151 \pm 2\mu$s.

2 Introduction & Theory

2.1 High and Low pass Filters

A High Pass Filter is an electric circuit which allows high frequency signals to pass, while reducing the amplitude of frequencies lower than the filters cut-off frequency. Similarly, a Low Pass Filter allows low frequencies to pass while reducing the amplitude of high frequencies.

A high pass filter consists of a voltage placed across a capacitor and resistor in series.

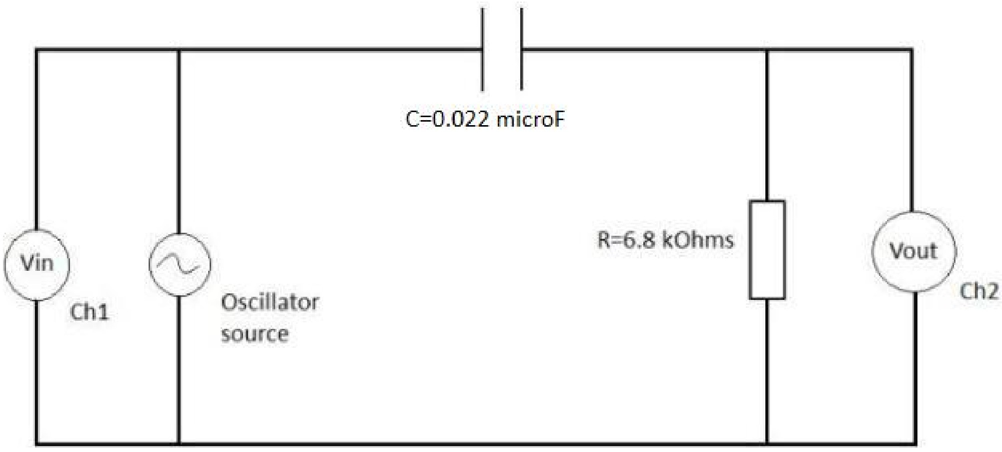

Figure 1: The High Pass Filter network

A low pass filter however consists of a voltage source and resistor in series with a capacitor in parallel. The capacitor causes a reactance which blocks low frequencies, causing them to go to the output voltage instead, which at high frequencies the reactance drops and the capacitor acts as a short circuit.

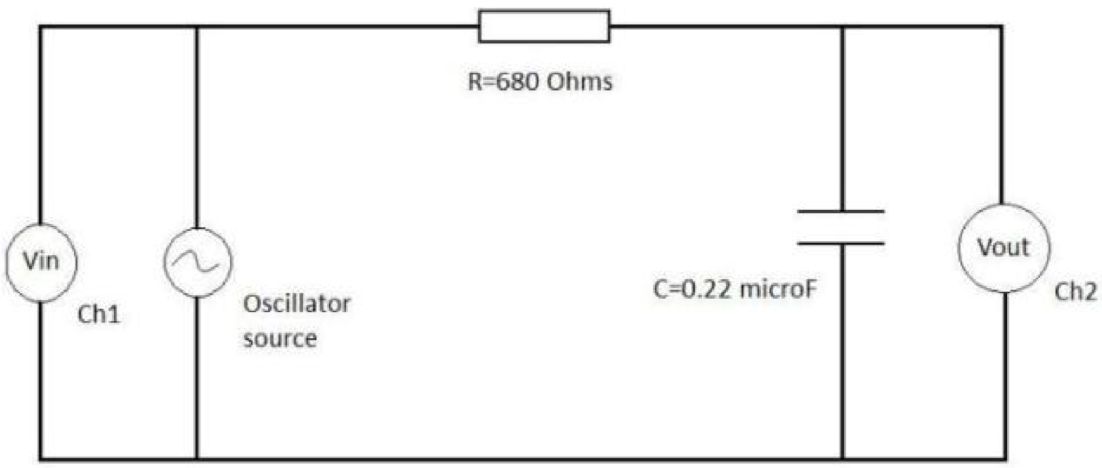

Figure 2: The Low Pass Filter network

We get the following relation between the Resistance, R, the Capacitance, C, and the *Cut-off Frequency*, f_c, for a filter network

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi\tau} \tag{1}$$

where τ is the time constant of the network.

2.2 Half Power Point

The Half Power Point of a filter network is the frequency at which the output power is half the input power. This occurs when the output voltage has dropped by a factor of $\sqrt{2} \approx 0.707$. This is also called the 3dB point, as it occurs when the log of the frequency is 3.

2.3 Time Constant

The Time Constant, τ, is the main characteristic of a filter network given by

$$\tau = RC \tag{2}$$

where R is the Resistance, and C is the Capacitance of the network. After a time of τ the voltage will fall to $1 - e^{-1} \approx 37\%$.

3 Experimental Method

3.1 The High Pass Filter

The high pass filter network was set up in the manor described above using an oscillator as the frequency and input voltage source. An oscilloscope was used to monitor the amplitudes of the input and output voltages, V_{in} and V_{out} simultaneously.

The earth of the oscilloscope and that of the oscillator were connected together.

The region over which V_{out} has the most change was found. The peek-to-peek output voltage, V_{out}, was measured for a range of 30-40 different frequencies, f, while the input voltage, V_{in} was kept constant.

The time delay, t, between the two signals was also measured for the same frequencies. The Phase Differences were then calculated from these using the relation

$$\phi = 2\pi f t \tag{3}$$

A graph of $\log_{10}(V_{out}/V_{in})$ versus $\log_{10}(f)$ was and also a graph of ϕ versus $\log_{10}(f)$ were plotted.

The half-power point of the filter was found using the former graph.

The value of ϕ at the half power point was found using the latter graph.

3.2 The Low Pass Filter

The circuit was changed to act as a low pass filter as described above.

The changes in V_{out} as f was varied were recorded.

Finally, the half-power-point was measured, and ϕ at this frequency.

3.3 Square Wave Response

The circuit was changed to act as a high pass filter again, and the oscillator was set to give a square wave signal.

The wave forms were printed for three different frequencies such that

$$\frac{1}{f} \ll RC \ , \ \frac{1}{f} = 2RC \ , \ \frac{1}{f} \gg RC$$

The time constant was then measured.

The circuit was then changed back to a low pass filter, and the above repeated.

4 Results & Analysis

4.1 The High Pass Filter

For the high pass filter the input voltage, V_{in}, was set to a constant value of 12V. The network had a capacitance of $C = 0.022\mu$F and a resistance of $R = 6.8$kΩ. The following data was obtained, the data for the time delays then used to calculate the phase differences with equation (3), and the graphs were plotted using this data

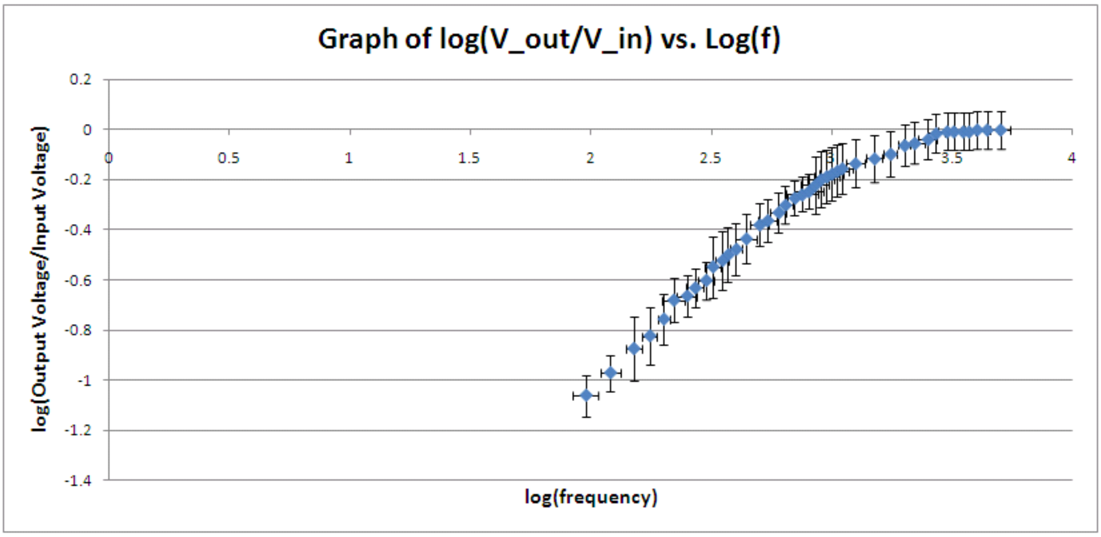

Figure 3: Log Graph of the ratio of output to input voltage versus the frequency

Frequency f (Hz)	Output Voltage V_{out} (V)	Time Delay t (μs)	Phase Difference ϕ
95±5	1.04±0.08	2600±100	1.55±0.10
120±5	1.28±0.08	2000±100	1.51±0.10
150±5	1.6±0.2	1500±100	1.41±0.11
175±5	1.8±0.2	1300±50	1.43±0.07
200±5	2.1±0.2	1100±50	1.38±0.07
220±10	2.5±0.2	1000±50	1.38±0.09
250±10	2.6±0.2	850±50	1.34±0.09
270±10	2.8±0.2	750±50	1.27±0.10
300±10	3.0±0.2	675±50	1.27±0.10
320±10	3.4±0.4	650±50	1.31±0.11
350±10	3.6±0.4	575±50	1.26±0.12
370±10	3.8±0.4	550±20	1.28±0.05
400±10	4.0±0.4	480±20	1.21±0.06
440±20	4.4±0.4	420±20	1.16±0.08
500±20	5.0±0.4	360±20	1.13±0.08
540±20	5.2±0.4	320±20	1.09±0.08
600±20	5.6±0.4	280±20	1.06±0.08
640±20	6.0±0.4	260±20	1.05±0.09
700±20	6.4±0.4	230±10	1.01±0.05
750±50	6.6±0.4	200±10	0.94±0.08
800±50	6.8±0.4	180±10	0.90±0.08
850±50	7.2±0.8	170±10	0.91±0.08
900±50	7.6±0.8	150±10	0.85±0.07
950±50	7.8±0.8	140±10	0.84±0.07
1000±50	8.0±0.8	130±10	0.82±0.07
1050±50	8.2±0.8	110±10	0.73±0.07
1100±50	8.4±0.8	100±10	0.69±0.08
1250±50	8.8±0.8	95±5	0.75±0.05
1500±50	9.2±0.8	70±5	0.66±0.05
1750±50	9.6±0.8	50±5	0.55±0.06
2000±50	10.4±0.8	40±2	0.50±0.03
2200±100	10.6±0.8	32±2	0.44±0.03
2500±100	11.0±0.8	24±2	0.38±0.03
2700±100	11.6±0.8	22±1	0.37±0.02
3000±100	11.8±0.8	18±1	0.34±0.02
3200±100	11.8±0.8	16±1	0.32±0.02
3500±100	11.8±0.8	13±1	0.29±0.02
3700±100	11.8±0.8	12±1	0.28±0.02
4000±200	12.0±0.8	11±1	0.28±0.03
4400±200	12.0±0.8	8±1	0.22±0.03
5000±200	12.0±0.8	6±1	0.19±0.03

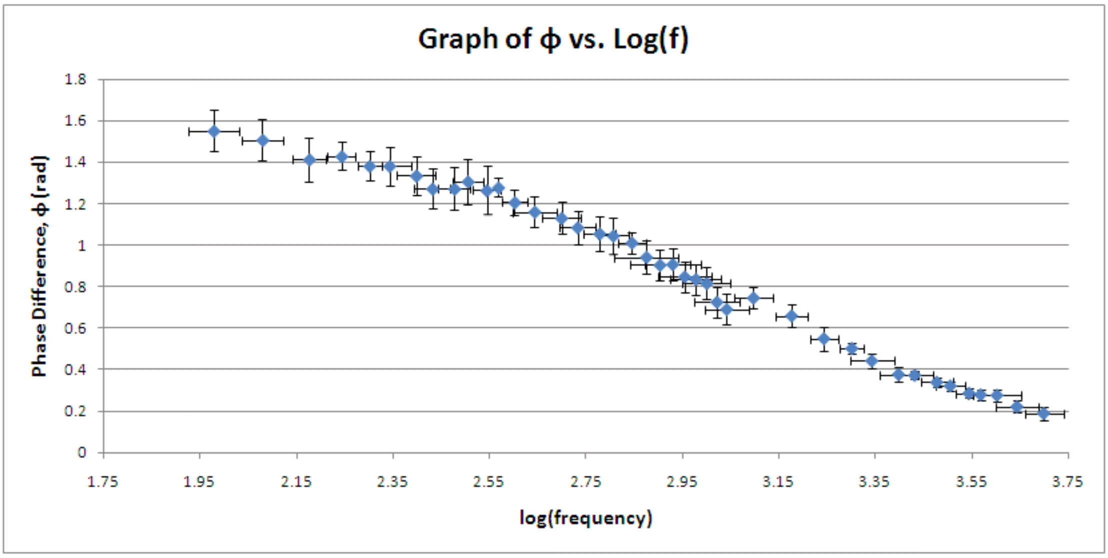

Figure 4: Graph of the Phase Difference versus the log of the frequency

4.2 The Low Pass Filter

For the low pass filter the input voltage was again set to 12V, and the network this time had a capacitance of $C = 0.22\mu$F and a resistance of $R = 680\Omega$.The following data was then obtained

Frequency f (Hz)	Output Voltage V_{out} (V)	Time Delay t (μs)	Phase Difference ϕ
10±0.5	12±0.8	0±50	0±0
100±5	12±0.8	150±50	0.09±0.03
1,000±50	8.8±0.8	120±10	0.75±0.07
10,000±500	1.28±0.08	23±1	1.45±0.10
100,000±5,000	0.26±0.02	7±1	4.40±0.67

4.3 Square Wave Response

When the oscillator was set to a square wave pulse the attached waveforms were obtained and printed for frequencies of 133Hz, 13,300Hz and 134,000Hz

The time constant was measured to be $\tau = 151 \pm 2\mu$s.

5 Error Analysis

The errors in the frequency, f, were taken from the small markings on the oscillator, while the errors in the input voltage, V_{in}, the output voltage, V_{out}, and time delay, t, were taken from the small markings on the oscilloscope.

For the calculated values the following equations were used

$$\Delta\log_{10}(f) = \frac{1}{f}\Delta f$$

$$\Delta\left(\frac{V_{out}}{V_{in}}\right) = \frac{V_{out}}{V_{in}} \times \sqrt{\left(\frac{\Delta V_{in}}{V_{in}}\right)^2 + \left(\frac{\Delta V_{out}}{V_{out}}\right)^2}$$

$$\Delta\log_{10}\left(\frac{V_{out}}{V_{in}}\right) = \frac{1}{V_{out}/V_{in}}\Delta\left(\frac{V_{out}}{V_{in}}\right)$$

$$\Delta\phi = \phi \times \sqrt{\left(\frac{\Delta t}{t}\right)^2 + \left(\frac{\Delta f}{f}\right)^2}$$

6 Conclusions

From the experiment we were able to see how high pass and low pass filters work. We found that they can reduce the amplitude of frequencies which are lower or higher than the cut-off frequencies. This caused the output voltage to most closely match the input voltage for a varying frequency source.

The half-power point for the high pass filter was found to be 1100 ± 50, corresponding to a value of $\log_{10} f_{high} = 3.04 \pm 0.5$. This is also know as the 3dB point, because it corresponds to a log of the frequency of 3. The value of the phase difference at the half point was found to be 0.69 ± 0.08rad. These are both within the limits of experimental error of the accepted values.

In the graphs log plots were used. This is of great advantage because we can see the change in the the ratio of the output to the input voltages over a very large range of frequencies.

For the low pass filter we saw that for higher frequencies the ratio of the output to the input voltages were approximately zero, while for lower frequencies the ratio was almost equal to one. This shows that the low pass filter was able to block out those frequencies that were above it's cut-off frequency. The half power point was measured to be 940 ± 5Hz, which had a phase difference of 0.71 ± 0.12rad.

Finally, when the oscillator was set to give square wave pulses the time constant was measured to be $\tau = 151 \pm 2\mu$s. This is in agreement with the theoretically calculated value of 149.6μs within the limits of experimental error. The RC filter circuits were also investigated as simple integrating and differentiating circuits.