The Non-Linear Pendulum

David-Alexander Robinson
08332461

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Contents

1 Abstract

In this experiment we used a pre-programed C program which used the *Simple Euler Method* to calculate the position and angular velocity of a Non-Linear pendulum, and write the date to a file, for a range of initial positions. We then used Gnuplot to plot graphs of the position and angular velocity with respect to time using this data.

Following this we altered the program to solve a Damped-Driven Non-Linear pendulum using the *Runge-Kutta Method*. Again, we used the data to plot graphs of the position and angular velocity of the pendulum with respect to time. We also plotted graphs of the phase space of the pendulum for varing values of initial amplitude.

2 Introduction and Theory

2.1 The Non-Linear Pendulum

In mechainics the equation of motion of a pendulum in a gravitational field is of the form

$$\frac{d^2s}{dt^2} = -\frac{g}{l}\sin(\theta)$$

This is the Non-Linear Pendulum equation.

For a small angels we have $\sin(\theta) \simeq \theta$ and we approximate the above to give the linear pendulum equation

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta$$

This equation is solvable and we get the usual perodic solution

$$\theta = a\sin(\omega t + \phi)$$

2.2 The Damped-Driven Pendulum

In a real enviorment we usually have some friction force which is directly proportional to the angular velocity ω of the pendulum, say

$$F_{fr} = -m\kappa\omega$$

which gives equation of motion

$$\frac{d^2\theta}{dt^2} - \kappa\frac{d\theta}{dt} = -\frac{g}{l}\sin(\theta)$$

If then, the system is acted on by some external driving force $mA\cos(\Omega t)$, say, we get a Damped-Driven pendulum which has equation of motion

$$\frac{d^2\theta}{dt^2} - \kappa\frac{d\theta}{dt} + \beta^2\sin(\theta) = A\cos(\Omega t)$$

3 Experimental Method

3.1 The Non-Linear Pendulum

The program "pendulum.c" was alltered to solve a linear pendulum. It was then run to calculate the position, θ, and angular velocity, ω, of the pendulum for a given number of iteritations. Gnupolt was then used to plot a graph of θ and ω versus time, t on the same graph. This was repeated for a range of initial positions.

The program was then again altered to solve a non-linear pendulum. As above, the program was run to calculate the data for the same initial positions. Gnuplot was then used to plot graphs of the linear values for θ and the non-linear values for θ versus t, again on the same graph.

3.2 The Damped-Driven Pendulum

The program was edited so that it would use the fourth order Runge-Kutta algorithm in calculating the values for θ and ω.

The program was run with values of zero for the damping coefficent and driving amplitude so that the data could be compared to that of the non-linear pendulum.

The program was then run with a value of 0.5 for the damping coefficent, and zero for the driving amplitude to calculate the data for the Damped Pendulum, using the same range of initial positions. Gnuplot was then used to plot graphs of the linear values of θ from above and the damped values of θ versus t.

The program was then edited again so that θ was restricted to values of $-\pi$ to π by including an if statement in the for loop. Also, an if statement was added before the $fprintf$ statement to stop the program from writing the first 5000 data points to the file, in order to let the system settle into perodic motion. It was then run again, this time with a range of values of driving amplitude and the same damping coefficent to solve the damped-driven pendulum. Again, Gnuplot was used to plot graphs of θ and ω versus t on the same graph.

Finally, Gnuplot was used to plot graphs of the phase space of the damped-driven non-linear pendulum, that is, graphs of θ versus ω.

4 Experimental Results

4.1 The Non-Linear Pendulum

For the data for obtained for the Non-Linear Pendulum the follow graphs showing θ and ω versus t were plotted using initial positions of $\theta = 0.2rad$, $1.0rad$ and $3.124rad$ respectivly.

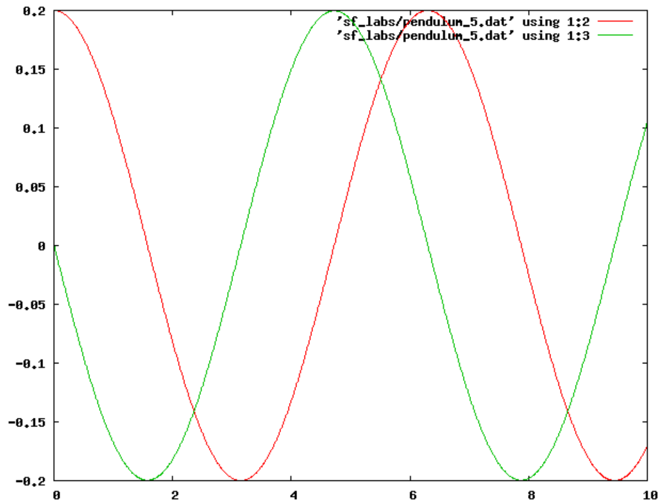

Figure 1: Initial angle of $\theta = 0.2rad$

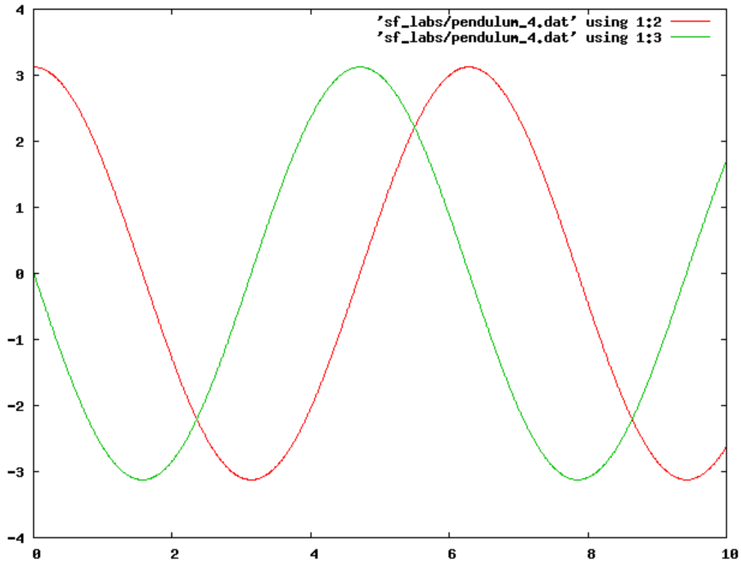

Figure 2: Initial angle of $\theta = 1.0 rad$

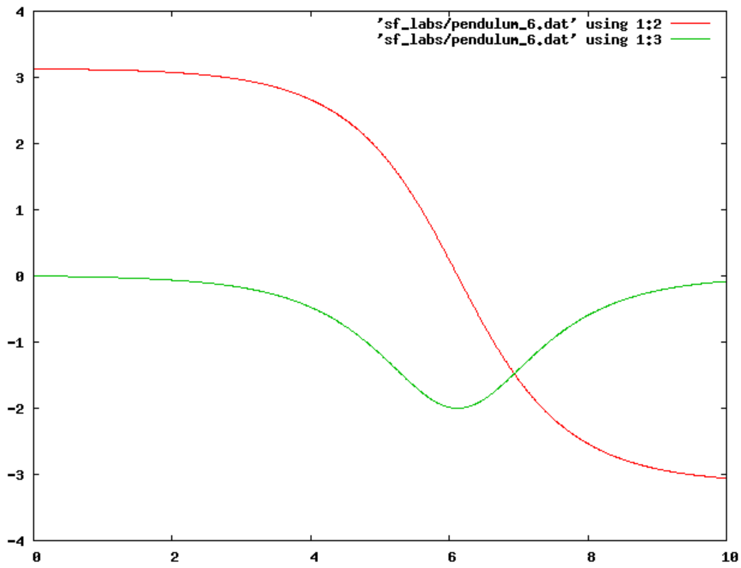

Figure 3: Initial angle of $\theta = 3.124 rad$

These graphs show a plot of the linear (red) versus the non-linear (green) pendulum using the same initial positions of $\theta = 0.2 rad$, $1.0 rad$ and $3.124 rad$ respectivly.

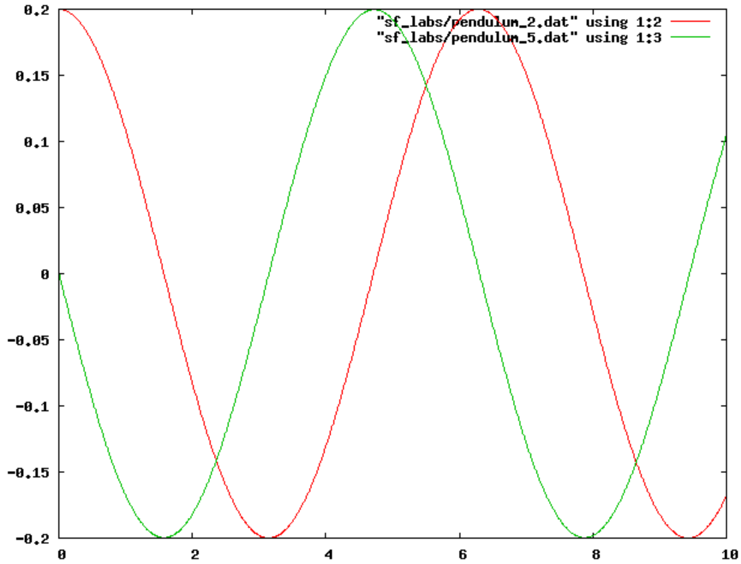

Figure 4: Initial angle of $\theta = 0.2rad$

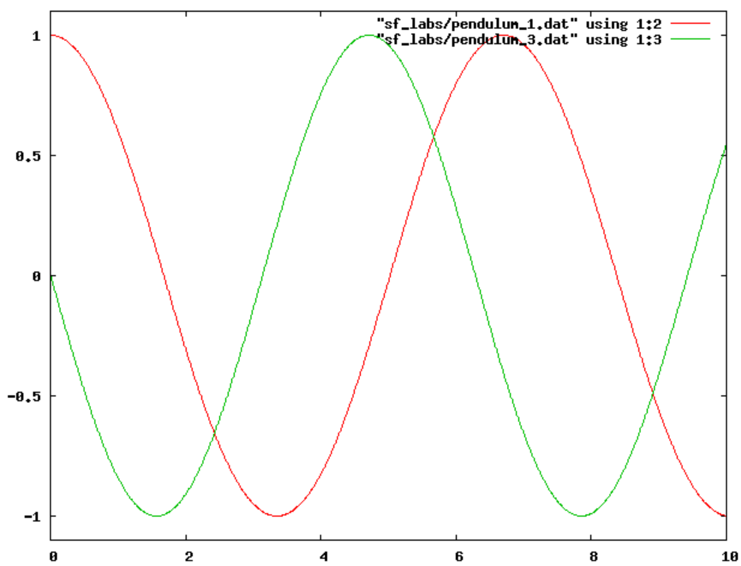

Figure 5: Initial angle of $\theta = 1.0rad$

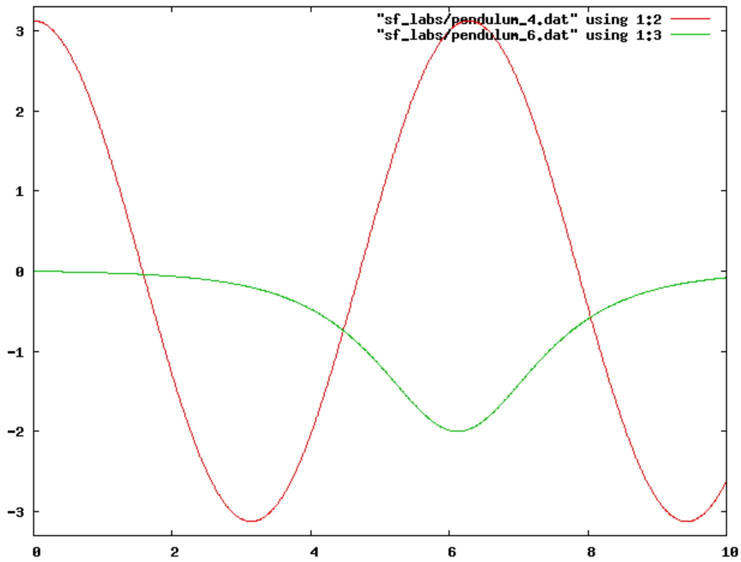

Figure 6: Initial angle of $\theta = 3.124rad$

4.2 The Damped-Driven Pendulum

For the damped pendulum, without a driving force, data was obatined that gave the following graphs, using the same values for intial position as above, and letting $\beta = 1$ and $\kappa = 0.5$ where

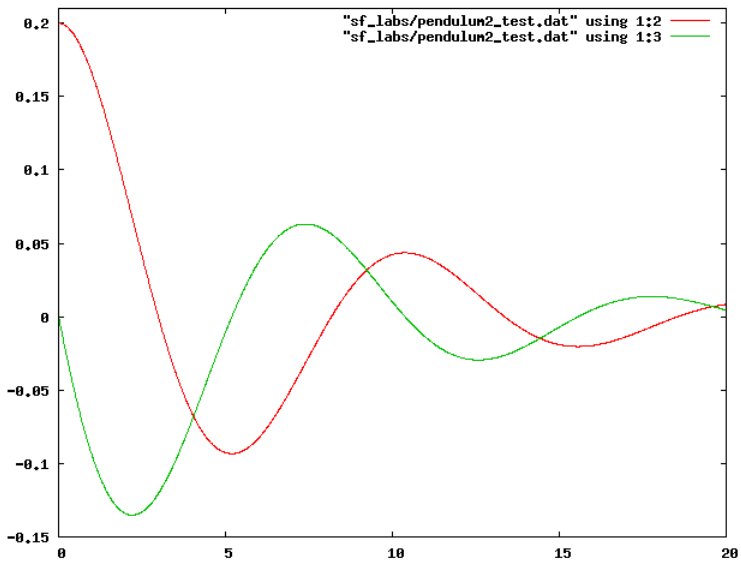

Figure 7: Initial angle of $\theta = 0.2rad$

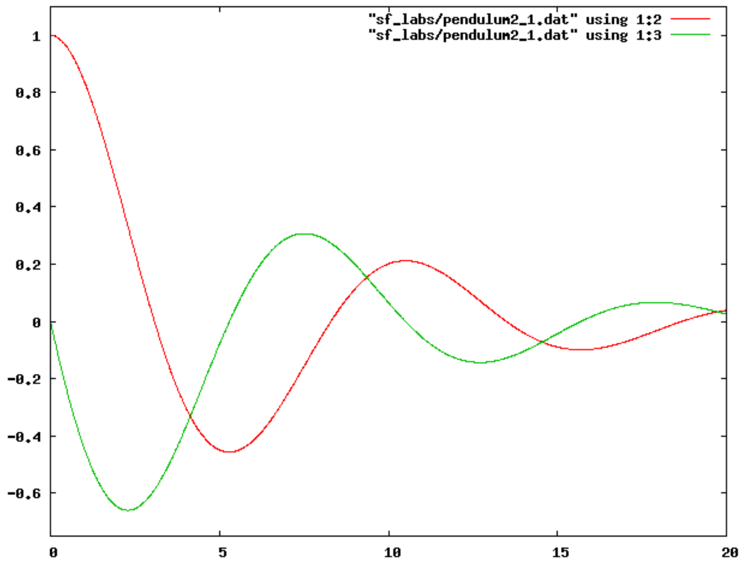

Figure 8: Initial angle of $\theta = 1.0 rad$

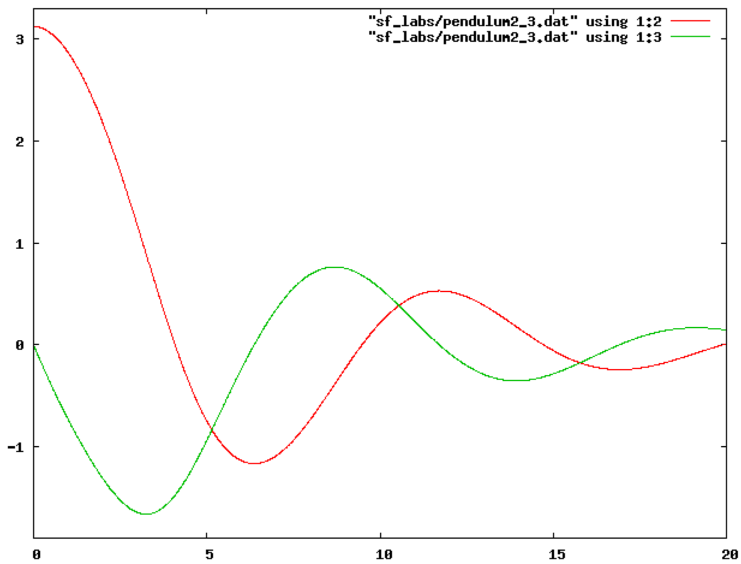

Figure 9: Initial angle of $\theta = 3.124 rad$

On changing the function to solve the damped-driven pendulum, with $\Omega = 0.6667$, different data was obtained, and the following graphs were plotted for the range of driving amplitudes $A = 0.90$, 1.07, 1.35, 1.47 and 1.5

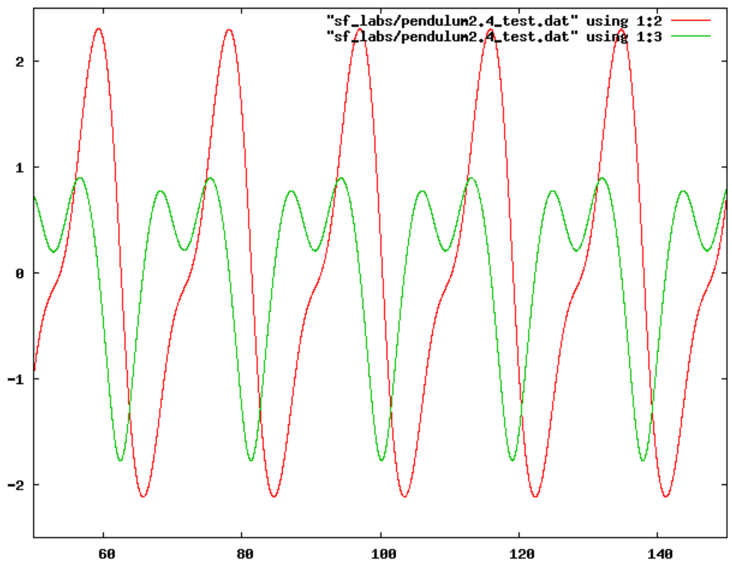

Figure 10: Driving Amplitude of $A = 0.90$

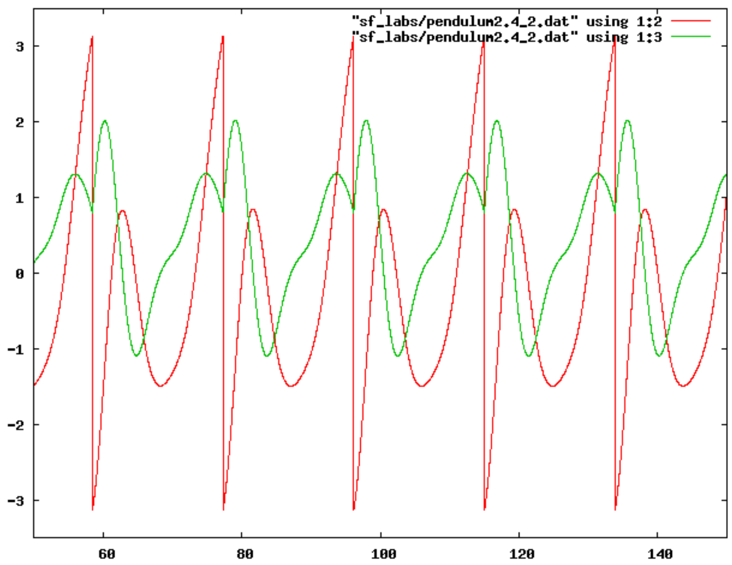

Figure 11: Driving Amplitude of $A = 1.07$

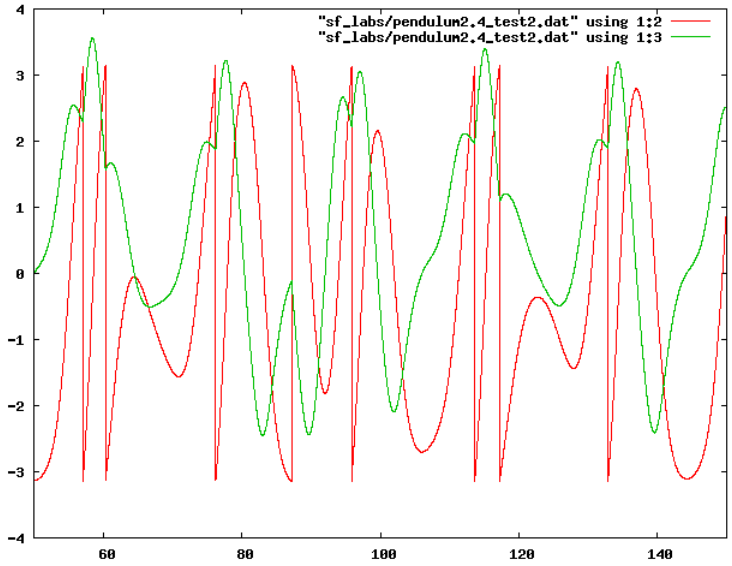

Figure 12: Driving Amplitude of $A = 1.35$

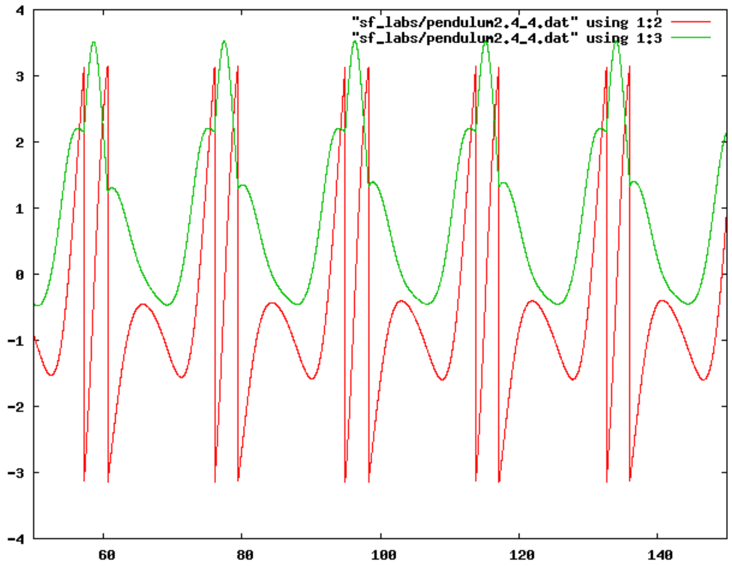

Figure 13: Driving Amplitude of $A = 1.47$

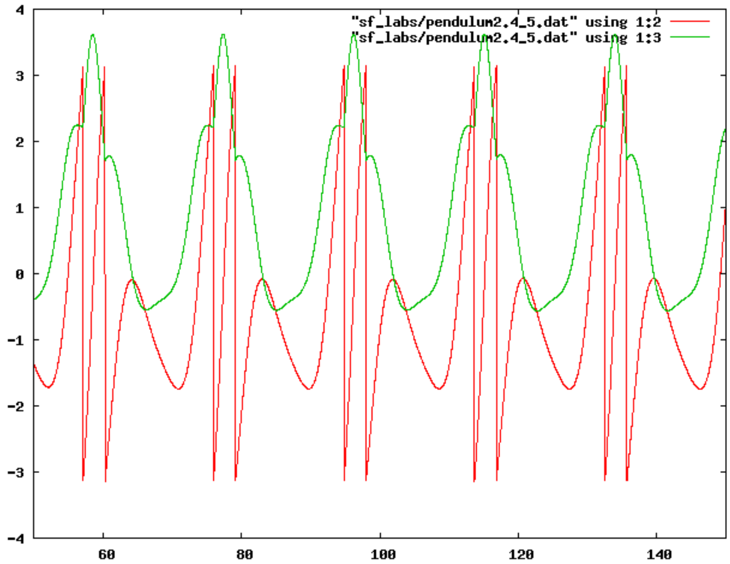

Figure 14: Driving Amplitude of $A = 1.5$

Finally, the Phase-Space diagrams corresponding to each of the above driving amplitudes were plotted

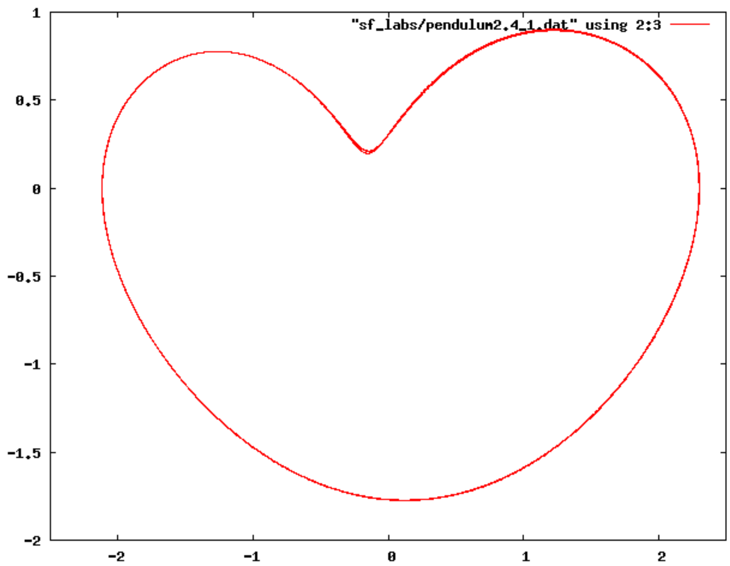

Figure 15: Driving Amplitude of $A = 0.90$

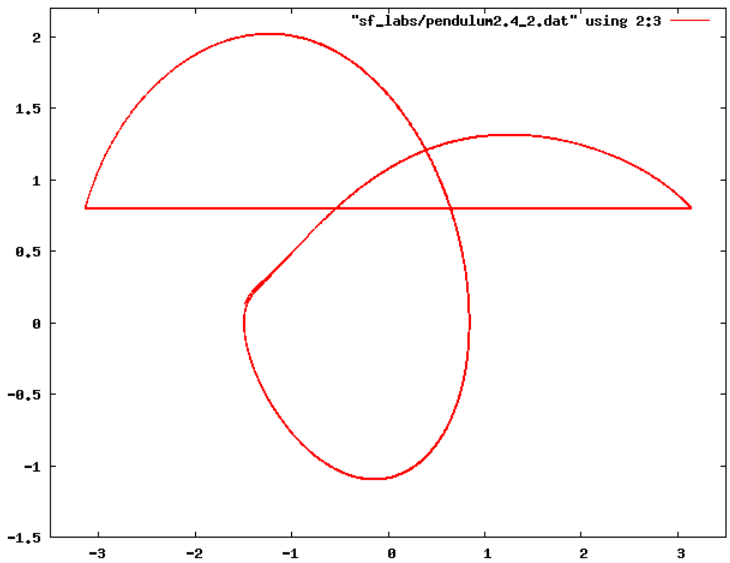

Figure 16: Driving Amplitude of $A = 1.07$

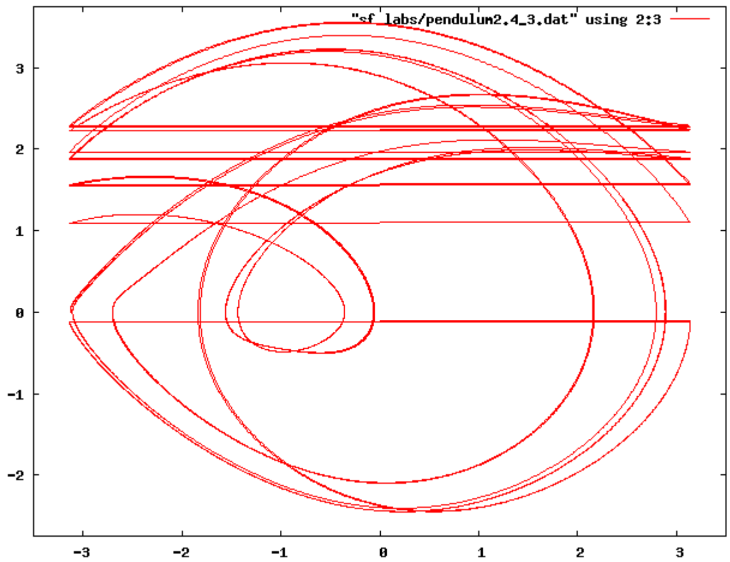

Figure 17: Driving Amplitude of $A = 1.35$

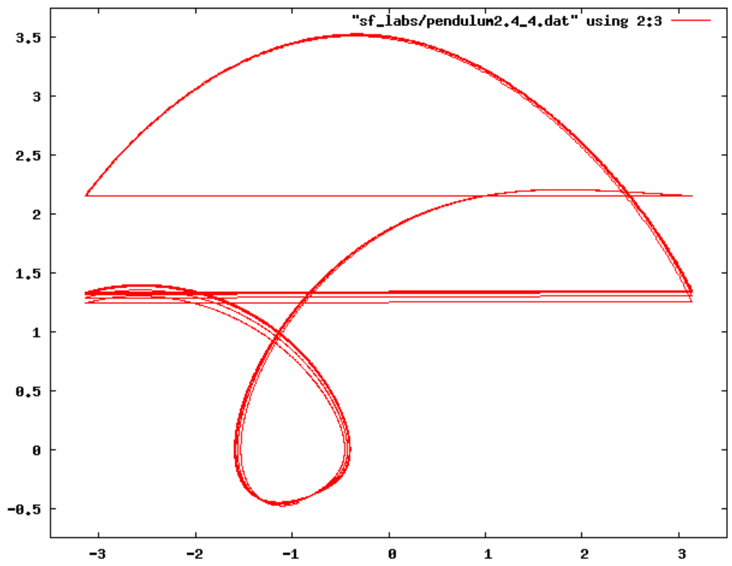

Figure 18: Driving Amplitude of $A = 1.47$

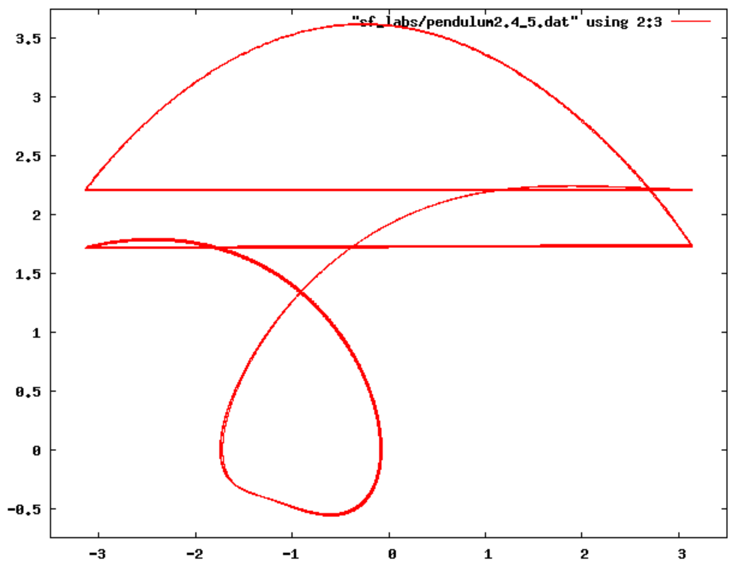

Figure 19: Driving Amplitude of $A = 1.5$

5 Error Analysis

There are errors in our calculations due to the rounding that we allow the computer to do. We were able to set the timestep of the program to a smaller value, but this will inhearently have a larger error due to the extra number of times that a number was rounded in obtaining the same overall length of the trajectory. In general we have

$$\Delta x \propto N$$

where N is the number of steps taken.

6 Conclusions

In our plots showing θ and ω versus t, we see that the pendulum preforms harmonic oscilliations for a small initial angel, however, as the angle increased to 3.124rad, the motion is clearly no longer simple harmonic. In the second set of graphs, we can see how θ for the non-linear pendulum varies from θ for the linearised pendulum. We conclude that the approximation of linearising the pendulum equation is good for a small initial angel, eg 0.2rad, but that the approximation breaks down for large initial angels, eg 3.124rad.

For the damped pendulum, our graphs show that the pendulum started with an initial amplitude equal to it's initial position, however as time progressed this amplitude decreased with an exponetial decay, and eventually stopped altogether.

In the next set of graphs we can see damped-driven pendulum motion. We see that for a small driving amplitude of A=0.90 the pendulum oscillates and we observe period-doubling. As the driving amplitude inreases the perodic motion starts to break down. For a driving amplitude of A=1.35 we get chaotic motion.

Finally, in our graphs of the phase space we can see the same observations as the above graphs. We see nice harmonic ossciallatory motion for the driving amplitude of A=0.90, and the pendulum preforms a "heart-shaped" trajectory. However, as the driving amplitude increases the perodicity starts to break down, and again when A=1.35 we see that the motion is chaotic.