Fourier Analysis

David-Alexander Robinson[*]; Daniel Tanner; Jack Denning
08332461

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1 Abstract

In this experiment an Oscillator, and the Cassy Lab program were used to investigate Fourier Transforms. Our aim was to attain an intuitive understanding of Fourier Analysis. The Oscillator was set up using the Cassy lab apparatus to display the sine wave functions on the computer. We were than able to analysis them using the software. We also analyzed the beat frequency of tuning forks, and investigated the frequencies of different vowels. The distribution of the harmonics were found to agree with the mathematical formula in all cases.

2 Introduction & Theory

Fourier Analysis is the process by which we mathematically approximate a periodic function, ie a function, say $f(x) : f(x+L) = f(x)$ where L is the period, by setting it as the sum of trigonometric functions, sin and cos. For the function $f(x)$ we get

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

where we have the coefficients a_0, a_n and b_n given by

$$a_0 = \frac{1}{l}\int_{-l/2}^{l/2} f(x)dx$$

$$a_n = \frac{2}{l}\int_{-l/2}^{l/2} f(x)\cos\left(\frac{2\pi n}{l}x\right)dx$$

$$b_n = \frac{2}{l}\int_{-l/2}^{l/2} f(x)\sin\left(\frac{2\pi n}{l}x\right)dx$$

3 Experimental Method

The Cassylab program and equipment first must be set up and the correct settings chosen. There is a certain amount of adjustment required to the sampling rate before each measurement to insure that it does not alias the data. Aliasing occurs when the frequency ¿ , as there is less than 2 points taken per cycle and the signal will take an the form of a wave of a lower frequency. Therefore it is vital that this ratio is avoided whenever possible, though the minimum measuring interval is 10s so anything around and above 20kHz will begin to alias.

3.1 Experiment 1

An oscilloscope was attached to the Cassylab and set to produce a sin wave of appropriate frequency for the sampling rate. The sampling rate and x-value were adjusted until a perfect sin wave is shown. This was done for multiple sin waves of different frequencies.

3.2 Experiment 2

In this experiment the oscillator was connected to the speaker and the microphone connected to the Cassylab. It was then used to measure an audible square wave produced by the oscillator. We found the upper and lower thresholds of human hearing and the microphone. Using tuning forks, the microphone was used to measure a beat frequency. We also analyzed different voices and at different pitches.

3.3 Experiment 3

In this section a sine wave was produced using the oscillator and the frequency slowly increased until it moved off the scale. The change in the Fourier Transform was observed.

3.4 Experiment 4

A single rectangular pulse was recorded by, during recording, inserting and then swiftly removing the input into the input terminal. The Fourier Transform was analyzed and the first three minimum and maximum found and compared with theory.

3.5 Experiment 5

The Logic box was used to generate a square repetitive pulse, and it?s Fourier Transform analyzed. Similarly, a sine wave pulse was produced and analyzed using a similar technique as in Experiment 4.

3.6 Experiment 6

In this part, the Uncertainty Principle was investigated, using the fact that the Fourier Transform of a particle?s position is its momentum.

4 Results & Analysis

4.1 Experiment 1

The sine wave generated by the oscillator had the following form and Fourier Transform: As evident in the Fourier Transform above, the typical Fourier Transform of a sine wave is simply a single peak, due to the fact that it consists of only one frequency. This can be derived theoretically using De Moivre?s theorem in the following way: Where is defined as and and , or the Dirac delta function. is the peak of the Fourier Transform as mentioned above. The function:

describes a periodic square wave.

The function is odd, so we know that only the sine terms are non-zero. This makes calculating the harmonics much easier.

Where

Therefore: , , , etc..

Only the odd harmonics are present.

The wave generated at about 650-800 Hz follows this pattern with peaks at n=1, n=3 and so on:

4.2 Experiment 2

The spectrum of square waves in air was quite a bit different, this may be due to a combination of background noise and the wave interacting with air molecules as it travels from the speaker to the microphone. We found the upper and lower threshold of hearing to be about 19kHz and 1.5kHz respectively.The microphone could measure sounds as low as 200Hz, and was receiving a signal for sounds up into the 100kHz range, though this was not measured accurately due to aliasing on the equipment.

[Beat Frequency]

We didn?t compare a man?s and woman?s voice, though we did sample some high and low notes though singing and the results showed that we had an average frequency range of 300-700Hz.

4.3 Experiment 3

The sine wave was increased from about 200Hz to 30kHz. At around 20kHz aliasing started to occur and the graph began to show a lower frequency than the oscillator was putting out. This agreed with the expected value as the sampling rate was set to 10s.

4.4 Experiment 4

The rectangular wave pulse is given by:
And has a Fourier transform of the form:
In the lab, we found the min and max values to be as follows

f(0)	2.2
f(10)	0.475
f(17.1)	0.28

f(7)	0.07
f(14)	0.04
f(21)	0.03

From the Fourier Transform above, it is known that the minimum occur when and the maximum when it equals to 1.

4.5 Experiment 5

A square wave pulse is given by the formula :
The analysis of this wave is identical to that of the rectangular wave in part 4, but with V replaced by L .So the result follows Maxima Minima
Again the minima occur when=0 and the maxima occur when it =1. And we found again that our result agreed with this within experimental error

f(0)	4.2
f(3.66)	3.05
f(7.45)	0.1
f(2.32)	0
f(6.23)	0
f(13.55)	0

The sine wave pulse is given by

4.6 Experiment 6

At the quantum level, momentum becomes quantized in the form of , where is the wave vector. Through Schrdinger?s wave equation, the particle?s state can be described by . Therefore, when dealing on a quantum scale, momentum acts like a frequency.

5 Error Analysis

6 Conclusions