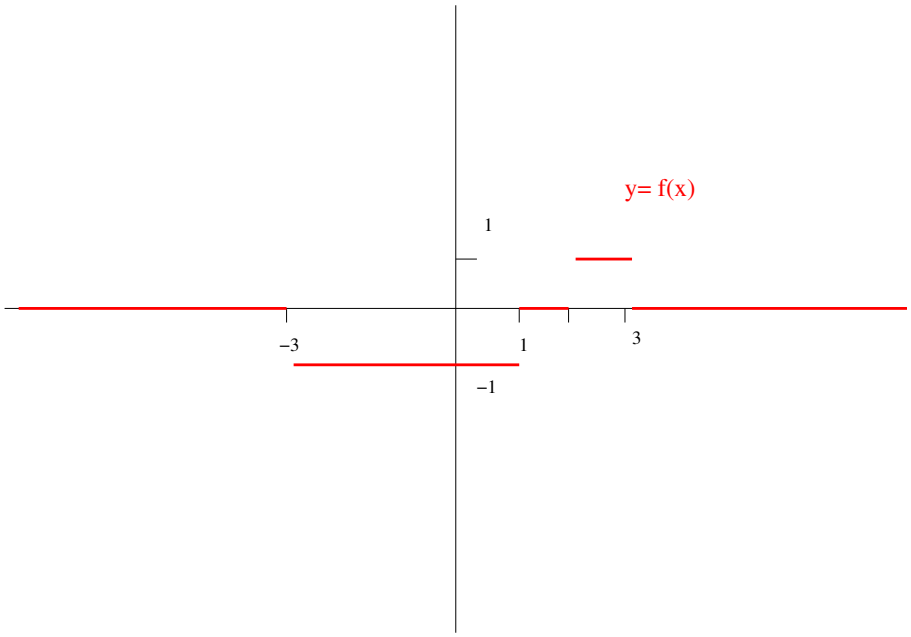
MA2224 (Lebesgue integral) Tutorial/excercise sheet 6
[issued March 16, 2018]

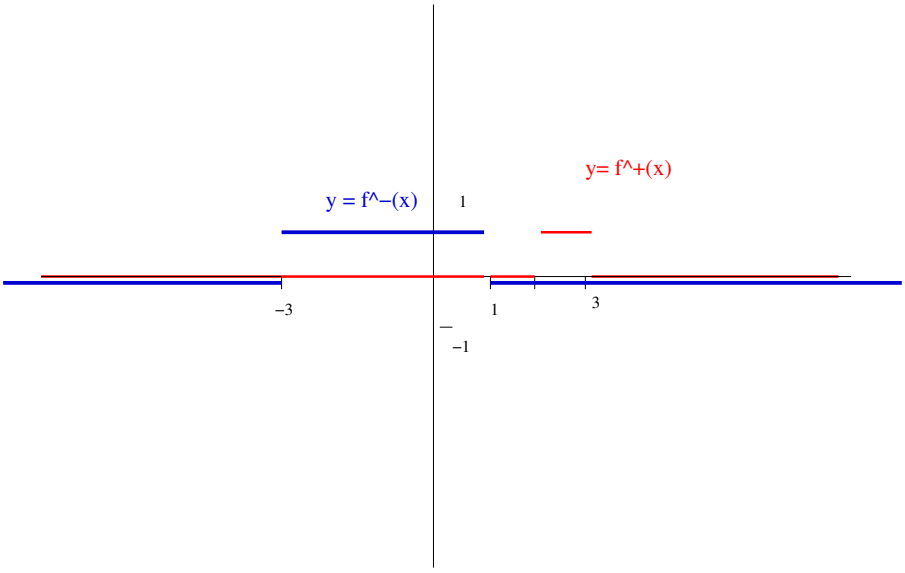
Name: Solutions

1. Define the sets
$$E_1 = [1, 3), \quad E_2 = (-3, 2)$$
(subsets of $\mathbb{R}$). Graph the simple function
$$y = f(x) = \chi_{E_1}(x) - \chi_{E_2}(x)$$
Solution:



and write it in its standard form (as a simple function).
Solution: The standard form is $f = (-1)\chi_{(-3,1)} + 1\chi_{[2,3)} + 0\chi_{(-\infty,-3]\cup[1,2]\cup(3,\infty)}$

2. Then, for the same f as in Q1, compute the functions f^+ and f^- (in their standard forms) and graph them.
Solution: Recall $f^+ = \max(f, 0)$ and $f^- = \max(-f, 0)$. So
$$f^+ = \chi_{[2,3)} = 1\chi_{[2,3)} + 0\chi_{\mathbb{R}\setminus[2,3)}, f^- = \chi_{(-3,1)} = 1\chi_{(-3,1)} + 0\chi_{\mathbb{R}\setminus(-3,1)},$$



3. Use the definition of the Lebesgue integral for nonnegative simple functions to compute $\int_{\mathbb{R}} f^+ \, d\mu$ and $\int_{[2,3]} f^+ \, d\mu$. (For the same f again. And apply the definitions directly.)

   Solution: Using the standard form for f^+,

$$\int_{\mathbb{R}} f^+ \, d\mu = 1\mu([2,3)) + 0\mu(\mathbb{R} \setminus [2,3)) = \mu([2,3) = 1.$$

   By definition $\int_{[2,3]} f^+ \, d\mu = \int_{\mathbb{R}} \chi_{[2,3]} f^+ \, d\mu$ and the standard form for $\chi_{[2,3]} f^+$ is $\chi_{[2,3]} f^+ = \chi_{[2,3)} = 1\chi_{[2,3)} + 0\chi_{\mathbb{R}\setminus[2,3)}$. So

$$\int_{[2,3]} f^+ \, d\mu = \int_{\mathbb{R}} \chi_{[2,3]} f^+ \, d\mu = 1\mu([2,3)) + 0\mu(\mathbb{R} \setminus [2,3)) = \mu([2,3)) = 1.$$

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