MA1S12 (Timoney) Tutorial sheet 9c

[March 26–31, 2014]

Name: Solution

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1. A loaded die has the following probabilities of showing the numbers 1–6 after a throw:

$$\frac{3}{15}, \frac{2}{15}, \frac{3}{15}, \frac{2}{15}, \frac{1}{15}, \frac{4}{15}$$

(in that order). Find the probability that one of the numbers $3, 4, 5$ will show after the die is thrown.

Solution:

$$P(3) + P(4) + P(5) = \frac{3}{15} + \frac{2}{15} + \frac{1}{15} = \frac{6}{15}$$

2. A random variable X associated with the outcome of tossing the same die as in the previous question has the values

$$X(1) = 0.9, X(2) = -2, X(3) = 1.1, X(4) = X(5) = 4, X(6) = 1.$$

Find the mean and also the standard deviation of the random variable.

Solution:

$$\begin{aligned}
\mu &= P(1)X(1) + P(2)X(2) + \cdots + P(6)X(6) \\
&= \frac{3}{15}(0.9) + \frac{2}{15}(-2) + \frac{3}{15}(1.1) + \frac{2}{15}4 + \frac{1}{15}4 + \frac{4}{15}1 \\
&= \frac{1}{15}(2.7 - 4 + 3.3 + 8 + 4 + 4) \\
&= \frac{18}{15} = 1.2
\end{aligned}$$

$$\begin{aligned}
\sigma^2 &= P(1)(X(1) - \mu)^2 + P(2)(X(2) - \mu)^2 + \cdots + P(6)(X(6) - \mu)^2 \\
&= \frac{3}{15}(0.9 - 1.2)^2 + \frac{2}{15}(-2 - 1.2)^2 + \frac{3}{15}(1.1 - 1.2)^2 \\
&\quad + \frac{2}{15}(4 - 1.2)^2 + \frac{1}{15}(4 - 1.2)^2 + \frac{4}{15}(1 - 1.2)^2 \\
&= 2.964 \\
\sigma &= 1.72163
\end{aligned}$$

3. A loaded coin has probability $p = 0.4$ of coming up heads. If the coin is tossed 10 times, find the probability that the number of heads will be either 4 or 5.

   Solution: This is an application of the binomial distribution ($n = 10$ independent trials, probability of success in each trial $= p = 0.4$ and probability of failure $= q = 1-p = 0.6$). What we want is

$$P(4) + P(5) = \binom{10}{4} p^4 q^{10-4} + \binom{10}{5} p^5 q^{10-5} = \binom{10}{4} p^4 q^6 + \binom{10}{5} p^5 q^5$$

   We have

$$\binom{10}{4} = \frac{10!}{4!6!} = \frac{(10)(9)(8)(7)(6)(5)(4)(3)(2)(1)}{(4!)(6)(5)(4)(3)(2)(1)} = \frac{(10)(9)(8)(7)}{4(3)(2)} = 210$$

   and

$$\binom{10}{5} = \frac{10!}{5!5!} = \frac{(10)(9)(8)(7)(6)(5)(4)(3)(2)(1)}{(5)(4)(3)(2)(1)5!} = \frac{(10)(9)(8)(7)(6)}{(5)(4)(3)(2)} = 252$$

   It works out that

$$P(4) + P(5) = 0.451481$$

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