MA1S12 (Timoney) Tutorial sheet 9a
[March 26–31, 2014]

Name: Solutions

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1. A loaded die has the following probabilities of showing the numbers 1–6 after a throw:

$$\frac{3}{15}, \frac{2}{15}, \frac{3}{15}, \frac{2}{15}, \frac{1}{15}, \frac{4}{15}$$

(in that order). Find the probability that a number ≥ 4 will show after the die is thrown.

Solution:

$$P(4) + P(5) + P(6) = \frac{2}{15} + \frac{1}{15} + \frac{4}{15} = \frac{7}{15}$$

2. A random variable X associated with the outcome of tossing the same die as in the previous question has the values

$$X(1) = 1, X(2) = -2, X(3) = 1.1, X(4) = X(5) = 4, X(6) = .9.$$

Find the mean and also the standard deviation of the random variable.

Solution:

$$\begin{aligned}
\mu &= P(1)X(1) + P(2)X(2) + \cdots + P(6)X(6) \\
&= \frac{3}{15}1 + \frac{2}{15}(-2) + \frac{3}{15}(1.1) + \frac{2}{15}4 + \frac{1}{15}4 + \frac{4}{15}(0.9) \\
&= \frac{1}{15}(3 - 4 + 3.3 + 8 + 4 + 3.6) \\
&= \frac{17.9}{15} = 1.19333
\end{aligned}$$

$$\begin{aligned}
\sigma^2 &= P(1)(X(1) - \mu)^2 + P(2)(X(2) - \mu)^2 + \cdots + P(6)(X(6) - \mu)^2 \\
&= \frac{3}{15}(1 - 1.19333)^2 + \frac{2}{15}(-2 - 1.19333)^2 + \frac{3}{15}(1.1 - 1.19333)^2 \\
&\quad + \frac{2}{15}(4 - 1.19333)^2 + \frac{1}{15}(4 - 1.19333)^2 + \frac{4}{15}(0.9 - 1.19333)^2 \\
&= 2.96729 \\
\sigma &= 1.72258
\end{aligned}$$

3. A loaded coin has probability $p = 0.495$ of coming up heads. If the coin is tossed 9 times, find the probability that the number of heads will be either 5 or 6.

Solution: This is an application of the binomial distribution ($n = 9$ independent trials, probability of success in each trial $= p = 0.495$ and probability of failure $= q = 1 - p = 0.505$). What we want is

$$P(5) + P(6) = \binom{9}{5} p^5 q^{9-5} + \binom{9}{6} p^6 q^{9-6} = \binom{9}{5} p^5 q^4 + \binom{9}{6} p^6 q^3$$

We have

$$\binom{9}{5} = \frac{9!}{5!4!} = \frac{9(8)(7)(6)(5)(4)(3)(2)(1)}{(5)(4)(3)(2)(1)4!} = \frac{9(8)(7)(6)}{4(3)(2)} = 126$$

and

$$\binom{9}{6} = \frac{9(8)(7)}{3!} = 84$$

It works out that

$$P(5) + P(6) = 0.402677$$

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