MA1S12 (Timoney) Tutorial sheet 7c
[March 10–14, 2014]

Name: Solutions

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1. Find the eigenvalues for the matrix

$$A = \begin{bmatrix} -1 & 4 \\ 2 & 4 \end{bmatrix}$$

Solution: We need to solve the chracteristic equation, $\det(A - \lambda I_2) = 0$ and that is

$$\det(A - \lambda I_2) = \det \begin{bmatrix} -1-\lambda & 4 \\ 2 & 4-\lambda \end{bmatrix} = (\lambda+1)(\lambda-4) - 8 = 0$$

or

$$\lambda^2 - 3\lambda - 12 = 0$$
$$\lambda = \frac{3 \pm \sqrt{3^2+48}}{2} = \frac{3 \pm \sqrt{57}}{2}$$

Eigenvalues are $(3+\sqrt{57})/2$ and $(3-\sqrt{57})/2$.

2. For

$$B = \begin{bmatrix} 3 & -2/3 \\ 0 & 2 \end{bmatrix}$$

it works out that $B = SDS^{-1}$ with

$$S = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}, D = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}, S^{-1} = \begin{bmatrix} 1 & -2/3 \\ 0 & 1/3 \end{bmatrix}.$$

(a) What are the eigenvalues of B and the corresponding eigenvectors?
Solution: The eigenavlues are the diagonal entries of D, that is 3 and 2.
The eigenvectors are the columns of S.
For $\lambda = 3$ we get the eigenvector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and for $\lambda = -3$ we get $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(b) Find e^{Bx} for $x \in \mathbb{R}$.
Solution: Since $Bx = S(Dx)S^{-1}$ (and Dx is diagonal) we have $e^{Bx} = Se^{Dx}S^{-1}$.

$$Dx = \begin{bmatrix} 3x & 0 \\ 0 & 2x \end{bmatrix}, \qquad e^{Dx} = \begin{bmatrix} e^{3x} & 0 \\ 0 & e^{2x} \end{bmatrix}$$

$$\begin{aligned} e^{Bx} &= Se^{Dx}S^{-1} = S \begin{bmatrix} e^{3x} & 0 \\ 0 & e^{2x} \end{bmatrix} \begin{bmatrix} 1 & -2/3 \\ 0 & 1/3 \end{bmatrix} \\ &= S \begin{bmatrix} e^{3x} & -2e^{3x}/3 \\ 0 & e^{2x}/3 \end{bmatrix} \\ &= \begin{bmatrix} e^{3x} & (-2e^{3x}+2e^{2x})/3 \\ 0 & e^{2x} \end{bmatrix} \end{aligned}$$

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