MA1S12 (Timoney) Tutorial sheet 7b
[March 10–14, 2014]

Name: Solutions

1. Find the eigenvalues for the matrix

$$A = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}$$

Solution: We need to solve the chracteristic equation, $\det(A - \lambda I_2) = 0$ and that is

$$\det(A - \lambda I_2) = \det \begin{bmatrix} -\lambda & 1 \\ 3 & 2-\lambda \end{bmatrix} = \lambda(\lambda - 2) - 3 = 0$$

or

$$\lambda^2 - 2\lambda - 3 = 0$$
$$(\lambda - 3)(\lambda + 1) = 0$$

Eigenvalues are 3 and -1.

2. For

$$B = \begin{bmatrix} -3 & 0 \\ 10/3 & 2 \end{bmatrix}$$

it works out that $B = SDS^{-1}$ with

$$S = \begin{bmatrix} 3 & 0 \\ -2 & 1 \end{bmatrix}, D = \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}, S^{-1} = \begin{bmatrix} 1/3 & 0 \\ 2/3 & 1 \end{bmatrix}.$$

(a) What are the eigenvalues of B and the corresponding eigenvectors?
Solution: The eigenavlues are the diagonal entries of D, that is -3 and 2.
The eigenvectors are the columns of S.
For $\lambda = -3$ we get the eigenvector $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ and for $\lambda = 2$ we get $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(b) Find e^{Bx} for $x \in \mathbb{R}$.
Solution: Since $Bx = S(Dx)S^{-1}$ (and Dx is diagonal) we have $e^{Bx} = Se^{Dx}S^{-1}$.

$$Dx = \begin{bmatrix} -3x & 0 \\ 0 & 2x \end{bmatrix}, \qquad e^{Dx} = \begin{bmatrix} e^{-3x} & 0 \\ 0 & e^{2x} \end{bmatrix}$$

$$\begin{aligned} e^{Bx} &= Se^{Dx}S^{-1} = S \begin{bmatrix} e^{-3x} & 0 \\ 0 & e^{2x} \end{bmatrix} \begin{bmatrix} 1/3 & 0 \\ 2/3 & 1 \end{bmatrix} \\ &= S \begin{bmatrix} e^{-3x}/3 & 0 \\ 2e^{-2x}/3 & e^{-2x} \end{bmatrix} \\ &= \begin{bmatrix} e^{-3x} & 0 \\ (-2e^{-3x} + 2e^{-2x})/3 & e^{-2x} \end{bmatrix} \end{aligned}$$

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