

MA1S12 (Timoney) Tutorial sheet 7a
[March 10–14, 2014]

Name: Solutions

1. Find the eigenvalues for the matrix

$$A = \begin{bmatrix} 0 & 4 \\ 3 & 1 \end{bmatrix}$$

Solution: We need to solve the characteristic equation, $\det(A - \lambda I_2) = 0$ and that is

$$\det(A - \lambda I_2) = \det \begin{bmatrix} -\lambda & 4 \\ 3 & 1 - \lambda \end{bmatrix} = \lambda(\lambda - 1) - 12 = 0$$

or

$$\begin{aligned} \lambda^2 - \lambda - 12 &= 0 \\ (\lambda - 4)(\lambda + 3) &= 0 \end{aligned}$$

Eigenvalues are 4 and -3 .

2. For

$$B = \begin{bmatrix} -3 & 10/3 \\ 0 & 2 \end{bmatrix}$$

it works out that $B = SDS^{-1}$ with

$$S = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}, D = \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}, S^{-1} = \begin{bmatrix} 1 & -2/3 \\ 0 & 1/3 \end{bmatrix}.$$

- (a) What are the eigenvalues of B and the corresponding eigenvectors?

Solution: The eigenvalues are the diagonal entries of D , that is -3 and 2 .

The eigenvectors are the columns of S .

For $\lambda = -3$ we get the eigenvector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and for $\lambda = 2$ we get $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

- (b) Find e^{Bx} for $x \in \mathbb{R}$.

Solution: Since $Bx = S(Dx)S^{-1}$ (and Dx is diagonal) we have $e^{Bx} = Se^{Dx}S^{-1}$.

$$Dx = \begin{bmatrix} -3x & 0 \\ 0 & 2x \end{bmatrix}, \quad e^{Dx} = \begin{bmatrix} e^{-3x} & 0 \\ 0 & e^{2x} \end{bmatrix}$$

$$\begin{aligned} e^{Bx} &= Se^{Dx}S^{-1} = S \begin{bmatrix} e^{-3x} & 0 \\ 0 & e^{2x} \end{bmatrix} \begin{bmatrix} 1 & -2/3 \\ 0 & 1/3 \end{bmatrix} \\ &= S \begin{bmatrix} e^{-3x} & -2e^{-3x}/3 \\ 0 & e^{2x}/3 \end{bmatrix} \\ &= \begin{bmatrix} e^{-3x} & (-2e^{-3x} + 2e^{2x})/3 \\ 0 & e^{2x} \end{bmatrix} \end{aligned}$$