MA1S12 (Timoney) Tutorial sheet 5b

[February 17–21, 2014]

Name: Solutions

For the first two questions, let

$$A = \begin{bmatrix} (1+4\sqrt{2})/9 & 2(1+\sqrt{2})/9 & 2(-1+2\sqrt{2})/9 \\ (2-4\sqrt{2})/9 & (8+5\sqrt{2})/18 & (-8+\sqrt{2})/18 \\ -2(1+\sqrt{2})/9 & (-8+7\sqrt{2})/18 & (8+5\sqrt{2})/18 \end{bmatrix}$$

Then A is in fact an orthogonal matrix and $\det(A) = 1$.

1. Show that the axis of rotation for A is parallel to the vector $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$.

   Solution:

   $$A \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} = \begin{bmatrix} \frac{1}{9}(1+4\sqrt{2}+4+4\sqrt{2}+4-8\sqrt{2}) \\ \frac{1}{18}(4-8\sqrt{2}+16+10\sqrt{2}+16-2\sqrt{2} \\ \frac{1}{18}(-4-4\sqrt{2}-16+14\sqrt{2}-16-10\sqrt{2} \end{bmatrix} = \begin{bmatrix} \frac{9}{9} \\ \frac{36}{18} \\ \frac{-36}{18} \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$$

   Since the vector $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ is fixed by A it must be along the axis of rotation.

2. Find $\cos\alpha$ where α is the angle of rotation for A.

   Solution: We know that $\operatorname{trace}(A) = 1 + 2\cos\alpha$. So we get

   $$1 + 2\cos\alpha = (1+4\sqrt{3})/9 + (8+5\sqrt{3})/18 + (8+5\sqrt{3})/18 = 1 + \sqrt{3}$$

   Thus $\cos\alpha = \sqrt{3}/2$.

3. Use the Gram-Schmidt method starting with the 3 vectors $\mathbf{u} = (\mathbf{2}i - \mathbf{j} + \mathbf{k})/\sqrt{6}$, $\mathbf{r} = \mathbf{j} + \mathbf{k}$ and $\mathbf{s} = \mathbf{k}$.

   Solution: See solution to sheet 5a (different vectors).

   Answer should be $\mathbf{u}$, $\mathbf{v} = (\mathbf{j} - \mathbf{k})/\sqrt{2}$ and $\mathbf{w} = (-\mathbf{i} - \mathbf{j} + \mathbf{k})/\sqrt{3}$

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