MA1S12 (Timoney) Tutorial sheet 3
[February 3–7, 2014]

Name: Solutions

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1. Find $\operatorname{adj}(A)$ for

$$A = \begin{bmatrix} 4 & 0 & 1 \\ 0 & -2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution: The cofactor matrix for A is

$$\begin{bmatrix} +\det\begin{bmatrix} -2 & 3 \\ 0 & 1 \end{bmatrix} & -\det\begin{bmatrix} 0 & 3 \\ 0 & 1 \end{bmatrix} & +\det\begin{bmatrix} 0 & -2 \\ 0 & 0 \end{bmatrix} \\ -\det\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} & +\det\begin{bmatrix} 4 & 1 \\ 0 & 1 \end{bmatrix} & -\det\begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix} \\ +\det\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} & -\det\begin{bmatrix} 4 & 1 \\ 0 & 3 \end{bmatrix} & +\det\begin{bmatrix} 4 & 0 \\ 0 & -2 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 4 & 0 \\ 2 & -12 & -8 \end{bmatrix}$$

$\operatorname{adj}(A)$ is the transpose of this cofactor matrix:

$$\operatorname{adj}(A) = \begin{bmatrix} -2 & 0 & -2 \\ 0 & 4 & -12 \\ 0 & 0 & -8 \end{bmatrix}$$

What are $\operatorname{adj}(A)A$ and $A\operatorname{adj}(A)$?

Solution: Both are $\det(A)I_3$. Since A is triangular $\det(A)$ is the product of the diagonal entries of A, which is -8. So both are $-8I_3 = \begin{bmatrix} -8 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & -8 \end{bmatrix}$.

2. Use Cramers rule to write a determinant formula for x_1 if (x_1, x_2, x_3) solves the system

$$\begin{cases} 2x_1 - x_2 = 5 \\ -x_1 + 4x_2 + x_3 = 2 \\ 6x_1 + 2x_2 - x_3 = 7 \end{cases}$$

Solution:

$$x_1 = \frac{\det\begin{bmatrix} 5 & -1 & 0 \\ 2 & 4 & 1 \\ 7 & 2 & -1 \end{bmatrix}}{\det\begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & 1 \\ 6 & 2 & -1 \end{bmatrix}}$$

3. Use the determinant method to find the equation of the circle in $\mathbb{R}^2$ that passes through the 3 points $(4, 5)$, $(7, 11)$ and $(5, 1)$.

Solution: The equation is

$$\det \begin{bmatrix} x^2 + y^2 & x & y & 1 \\ 4^2 + 5^2 & 4 & 5 & 1 \\ 7^2 + 11^2 & 7 & 11 & 1 \\ 5^2 + 1^2 & 5 & 1 & 1 \end{bmatrix} = 0$$

$$\begin{aligned}
\det \begin{bmatrix} x^2 + y^2 & x & y & 1 \\ 41 & 4 & 5 & 1 \\ 170 & 7 & 11 & 1 \\ 26 & 5 & 1 & 1 \end{bmatrix} &= (x^2 + y^2) \det \begin{bmatrix} 4 & 5 & 1 \\ 7 & 11 & 1 \\ 5 & 1 & 1 \end{bmatrix} - x \det \begin{bmatrix} 41 & 5 & 1 \\ 170 & 11 & 1 \\ 26 & 1 & 1 \end{bmatrix} \\
& \quad + y \det \begin{bmatrix} 41 & 4 & 1 \\ 170 & 7 & 1 \\ 26 & 5 & 1 \end{bmatrix} - \det \begin{bmatrix} 41 & 4 & 5 \\ 170 & 7 & 11 \\ 26 & 5 & 1 \end{bmatrix} \\
\det \begin{bmatrix} 4 & 5 & 1 \\ 7 & 11 & 1 \\ 5 & 1 & 1 \end{bmatrix} &= \det \begin{bmatrix} 4 & 5 & 1 \\ 3 & 6 & 0 \\ 1 & -4 & 0 \end{bmatrix} \\
&= 4(0) - 5(0) + 1(-12 - 6) = -18 \\
\det \begin{bmatrix} 41 & 5 & 1 \\ 170 & 11 & 1 \\ 26 & 1 & 1 \end{bmatrix} &= \det \begin{bmatrix} 41 & 5 & 1 \\ 129 & 6 & 0 \\ -15 & -4 & 0 \end{bmatrix} \\
&= 41(0) - 5(0) + 1(-516 + 90) = -426 \\
\det \begin{bmatrix} 41 & 4 & 1 \\ 170 & 7 & 1 \\ 26 & 5 & 1 \end{bmatrix} &= \det \begin{bmatrix} 41 & 4 & 1 \\ 129 & 3 & 0 \\ -15 & 1 & 0 \end{bmatrix} \\
&= 41(0) - 4(0) + 1(129 + 45) = 174 \\
\det \begin{bmatrix} 41 & 4 & 5 \\ 170 & 7 & 11 \\ 26 & 5 & 1 \end{bmatrix} &= 41(7 - 55) - 4(170 - 286) + 5(850 - 182) = 1836
\end{aligned}$$

Equation is

$$-18(x^2 + y^2) + 426x + 174y - 1836 = 0.$$

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