MA1S12 (Timoney) Tutorial sheet 1

[January 20–24, 2014]

Name: Solutions

1. For

$$A = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 4 & -2 \\ 8 & 9 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{bmatrix}$$

find $\det(A)$ using cofactor expansion along the first row.

Solution:

$$\begin{aligned} \det(A) &= 1 \times \det \begin{bmatrix} 1 & 4 & -2 \\ 9 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} - 0 + 3 \det \begin{bmatrix} 0 & 1 & -2 \\ 8 & 9 & 0 \\ 0 & 2 & 1 \end{bmatrix} + 0 \\ &= 1 \det \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - 4 \det \begin{bmatrix} 9 & 0 \\ 2 & 1 \end{bmatrix} + (-2) \det \begin{bmatrix} 9 & 1 \\ 2 & 0 \end{bmatrix} \\ &\quad +3 \left(0 - \det \begin{bmatrix} 8 & 0 \\ 0 & 1 \end{bmatrix} + (-2) \det \begin{bmatrix} 8 & 9 \\ 0 & 2 \end{bmatrix} \right) \\ &= 1 - 4(9) - 2(-2) + 3((-8) - 2(16)) \\ &= -151 \end{aligned}$$

2. For the same A, find $\det(A)$ via Gaussian elimination. (A different answer would be a surprise!)

Solution:

$$\begin{aligned} \det \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 4 & -2 \\ 8 & 9 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{bmatrix} &= \det \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 4 & -2 \\ 0 & 9 & -23 & 0 \\ 0 & 2 & 0 & 1 \end{bmatrix} \\ &\quad \text{(we have subtracted 8 times (old) Row 1 from Row 3)} \\ &= \det \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 4 & -2 \\ 0 & 0 & -59 & 18 \\ 0 & 0 & -8 & 1 \end{bmatrix} \\ &\quad \text{(we have subtracted 9 row 2 from row 3} \\ &\quad \text{and 2 row 2 from row 4)} \\ &= -59 \det \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 4 & -2 \\ 0 & 0 & 1 & -18/59 \\ 0 & 0 & -8 & 5 \end{bmatrix} \\ &\quad \text{(we have factored } -59 \text{ from row 3)} \end{aligned}$$

$$
\begin{aligned}
&= -59 \det \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 4 & -2 \\ 0 & 0 & 1 & -18/59 \\ 0 & 0 & 0 & 151/59 \end{bmatrix} \\
&\quad \text{(we have added } -8 \text{ row 3 to row 4)} \\
&= -59(151/59) = -151
\end{aligned}
$$

(we have used that the detminant of a triangular matrix is the product of the diagonal entries and not fully finished the Gaussian elimination).

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