MA1S12 (Timoney) Tutorial/exercise sheet 10
[March 31, 2014]

Name: Solutions

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1. The number of alpha particles hitting a certain detector per second is found to obey a Poisson distribution with mean 4. What then is the probabilty of 3, 4 or 5 alpha particles hitting the detector in a given second?

   Solution: The Poisson distribution gives probability

   $$P(i) = e^{-\mu}\frac{\mu^i}{i!}$$

   to each of the numbers $i = 0, 1, 2, \ldots$. Here we want $\mu = 4$ and the sum

   $$P(3) + P(4) + P(5) = e^{-4}\left(\frac{(4)^3}{3!} + \frac{(4)^4}{4!} + \frac{(4)^5}{5!}\right) = 0.547027.$$

   What is the probability that a number that is either ≤ 2 or ≥ 6 will hit in a given second?

   Solution: Since the otal probability is 1 (for all natural numbers $0, 1, 2, \ldots$) what we want is

   $$1 - (P(3) + P(4) + P(5)) = 1 - 0.547027 = 0.452973$$

   What is the standard deviation in this case?

   Solution: We have variance $\sigma^2 = \mu = 4$ for the Poisson distribution. So $\sigma = 2$ is the standard deviation.

2. A factory produces bottles of a soft drink that are sold as 2 litre bottles. A good model is that the quantity of liquid in a bottle obeys a normal distribution with mean 2.02 (litres) and standard deviation 0.09. What proportion of the bottles have less than 2 litres in them?

   Solution: We are supposing that the probability of $< x$ (litres in a bottle) is given by a normal distribution function

   $$\Phi_{\mu,\sigma}(x) = \Phi_{2.02,0.09}(x) = \Phi_{0,1}\left(\frac{x-\mu}{\sigma}\right) = \Phi_{0,1}\left(\frac{x-2.02}{0.09}\right)$$

   (in terms of the standard normal distribution $\Phi_{0,1}$ to be found in the tables). What we want is

   $$\Phi_{2.02,0.09}(2) = \Phi_{0,1}\left(\frac{2-2.02}{0.09}\right) = \Phi_{0,1}(-0.222222) = 1 - \Phi_{0,1}(0.222)$$

   (by symmetry of the standard normal). From the tables, $\Phi_{0,1}(0.22) = 0.5871$ while $\Phi_{0,1}(0.23) = 0.5910$ and we could guess that 0.222 would add about 2/10 of the difference, or about 0.0008, giving as our answer

   $$1 - 0.5879 = 0.4121$$

What proportion will have between 2 litres and 2.03 litres?

Solution: We have

$$P(\text{ range } [2, 2.03]) = P(< 2.03) - P(< 2)$$

and we have worked out $P(< 2) = 0.4121$ already.

For

$$P(< 2.03) = \Phi_{\mu,\sigma}(2.03) = \Phi_{2.02,0.09}(2.03) = \Phi_{0,1}\left(\frac{x-\mu}{\sigma}\right) = \Phi_{0,1}\left(\frac{2.03-2.02}{0.09}\right) = \Phi_{0,1}(0.111111)$$

that tables give 0.5438 for 0.11 and 0.5478 for 0.12. The difference $0.5478 - 0.5438 = 0.004$ divided by 9 is 0.000444444 and adjusting 0.5438 by this gives (about) 0.5482. So

$$P(\text{ range } [2, 2.03]) = P(< 2.03) - P(< 2) = 0.5482 - 0.4121 = 0.1361$$

3. Suppose these are 5 samples

1.16	1.12	1.09	0.98	1.28

from a normal distribution with mean μ. Use Student's t-distribution to give a symmetric 90% confidence interval for μ.

Solution:

Step 1. We have $\gamma = 0.90$.

Step 2. Then

$$\frac{1+\gamma}{2} = \frac{1.90}{2} = 0.95$$

For F the student t-distribution with $5 - 1 = 4$ degrees of freedom we find from the tables that $F(2.132) = 0.95$ (or actually that $1 - F(2.132) = 0.05$).

Step 3.

$$m = \frac{1.16 + 1.12 + 1.09 + 0.98 + 1.28}{5} = 1.126$$

$$s^2 = \frac{(1.16-m)^2 + (1.12-m)^2 + (1.09-m)^2 + (0.98-m)^2 + (1.28-m)^2}{4}$$

works out as 0.01188. So $s = \sqrt{0.01188} = 0.108995$.

Step 4. Then with $k = sc/\sqrt{n} = (0.108995)(2.132)/\sqrt{5} = 0.103922$ we get probability 0.9 that μ is in the range $m \pm 0.103922 = 1.126 \pm 0.103922$.

So in the interval $[1.02208, 1.22992]$ with confidence 0.9 (or 90%).

Unlike previous sheets, this one will not be graded. For those who have a tutorial left, you may wish to ask questions at the tutorial about these problems.

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