1S11 (Timoney) Tutorial sheet 7
[November 13 – 16, 2012]

Name:

Student ID:

Course: (Circle one) | JF Science | JF HG | JF NPCAM | JF CMM | JF MC |

1. Find all solutions of the following system of linear equations by using Gauss-Jordan elimination. (Follow the method exactly.)

$$\begin{array}{rcrcrcrcr} x_1 & + & 2x_2 & + & 4x_3 & - & 4x_4 & = & 5 \\ x_1 & + & 12x_2 & - & 6x_3 & - & 4x_4 & = & 15 \end{array}$$

Solution: The augmented matrix is

$$\left[\begin{array}{cccccc} 1 & 2 & 4 & -4 & : & 5 \\ 1 & 12 & -6 & -4 & : & 15 \end{array}\right]$$

$$\left[\begin{array}{cccccc} 1 & 2 & 4 & -4 & : & 5 \\ 0 & 10 & -10 & 0 & : & 10 \end{array}\right] \text{ OldRow2} - \text{OldRow1}$$

$$\left[\begin{array}{cccccc} 1 & 2 & 4 & -4 & : & 5 \\ 0 & 1 & -1 & 0 & : & 1 \end{array}\right] \text{ OldRow2} \times (1/10)$$

(That finishes Gaussian elimination. The matrix is in row echelon form. Now for the Gauss-Jordan part.)

$$\left[\begin{array}{cccccc} 1 & 0 & 6 & -4 & : & 3 \\ 0 & 1 & -1 & 0 & : & 1 \end{array}\right] \text{ OldRow1} - 2\text{OldRow2}$$

System of equations for this augmented matrix (which is in reduced row echelon form) is:

$$\left\{\begin{array}{rcrcrcrcr} x_1 & & & + & 6x_3 & - & 4x_4 & = & 3 \\ & & x_2 & - & x_3 & & & = & 1 \end{array}\right.$$

$$\left\{\begin{array}{rcl} x_1 & = & 3 - 6x_3 + 4x_4 \\ x_2 & = & 1 + x_3 \\ & & x_3 \text{ and } x_4 \text{ free} \end{array}\right.$$

2. Find all solutions of the following system of linear equations by using Gauss-Jordan elimination. (Follow the method exactly.)

$$\begin{array}{rcrcrcrcr} & & 5x_2 & + & 6x_3 & + & 6x_4 & = & 21 \\ x_1 & - & 2x_2 & + & x_3 & - & 4x_4 & = & -3 \\ x_1 & + & 3x_2 & + & 7x_3 & + & 2x_4 & = & 18 \\ x_1 & - & 12x_2 & - & 11x_3 & - & 8x_4 & = & -37 \end{array}$$

Solution: The augmented matrix is

$$\left[\begin{array}{cccccc} 0 & 5 & 6 & 6 & : & 21 \\ 1 & -2 & 1 & -4 & : & -3 \\ 1 & 3 & 7 & 2 & : & 18 \\ 1 & -12 & -11 & -8 & : & -37 \end{array}\right]$$

$$\left[\begin{array}{cccccc} 1 & -2 & 1 & -4 & : & -3 \\ 0 & 5 & 6 & 6 & : & 21 \\ 1 & 3 & 7 & 2 & : & 18 \\ 1 & -12 & -11 & -8 & : & -37 \end{array}\right] \begin{array}{l} \text{OldRow2} \\ \text{OldRow1} \\ \\ \end{array}$$

$$\left[\begin{array}{cccccc} 1 & -2 & 1 & -4 & : & -3 \\ 0 & 5 & 6 & 6 & : & 21 \\ 0 & 5 & 6 & 6 & : & 21 \\ 0 & -10 & -12 & -4 & : & -34 \end{array}\right] \begin{array}{l} \\ \\ \text{OldRow3} - \text{OldRow1} \\ \text{OldRow4} - \text{OldRow1} \end{array}$$

$$\left[\begin{array}{cccccc} 1 & -2 & 1 & -4 & : & -3 \\ 0 & 1 & \frac{6}{5} & \frac{6}{5} & : & \frac{21}{5} \\ 0 & 5 & 6 & 6 & : & 21 \\ 0 & -10 & -12 & -4 & : & -34 \end{array}\right] \begin{array}{l} \\ \text{OldRow2} \times \frac{1}{5} \\ \\ \\ \end{array}$$

$$\left[\begin{array}{cccccc} 1 & -2 & 1 & -4 & : & -3 \\ 0 & 1 & \frac{6}{5} & \frac{6}{5} & : & \frac{21}{5} \\ 0 & 0 & 0 & 0 & : & 0 \\ 0 & 0 & 0 & 8 & : & 8 \end{array}\right] \begin{array}{l} \\ \\ \text{OldRow3} - 5 \times \text{OldRow2} \\ \text{OldRow3} + 10 \times \text{OldRow2} \end{array}$$

$$\left[\begin{array}{cccccc} 1 & -2 & 1 & -4 & : & -3 \\ 0 & 1 & \frac{6}{5} & \frac{6}{5} & : & \frac{21}{5} \\ 0 & 0 & 0 & 8 & : & 8 \\ 0 & 0 & 0 & 0 & : & 0 \end{array}\right] \begin{array}{l} \\ \\ \text{OldRow4} \\ \text{OldRow3} \end{array}$$

$$\left[\begin{array}{cccccc} 1 & -2 & 1 & -4 & : & -3 \\ 0 & 1 & \frac{6}{5} & \frac{6}{5} & : & \frac{21}{5} \\ 0 & 0 & 0 & 1 & : & 1 \\ 0 & 0 & 0 & 0 & : & 0 \end{array}\right] \begin{array}{l} \\ \\ \text{OldRow3} \times \frac{1}{8} \\ \\ \end{array}$$

(That finishes Gaussian elimination. The matrix is in row echelon form. Now for the Gauss-Jordan part.)

$$\left[\begin{array}{ccccc|l} 1 & -2 & 1 & 0 & 1 & \text{OldRow1} + 4 \times \text{OldRow3} \\ 0 & 1 & \frac{6}{5} & 0 & 3 & \text{OldRow2} - \frac{6}{5} \times \text{OldRow3} \\ 0 & 0 & 0 & 1 & 1 & \\ 0 & 0 & 0 & 0 & 0 & \end{array}\right]$$

$$\left[\begin{array}{ccccc|l} 1 & 0 & \frac{17}{5} & 0 & 7 & \text{OldRow1} + 2 \times \text{OldRow2} \\ 0 & 1 & \frac{6}{5} & 0 & 3 & \\ 0 & 0 & 0 & 1 & 1 & \\ 0 & 0 & 0 & 0 & 0 & \end{array}\right]$$

System of equations for this augmented matrix (which is in reduced row echelon form) is:

$$\left\{\begin{array}{rcrcrcl} x_1 & & & + & \frac{17}{5}x_3 & & = 7 \\ & & x_2 & + & \frac{6}{5}x_3 & & = 3 \\ & & & & & x_4 & = 1 \\ & & & & & 0 & = 0 \end{array}\right.$$

$$\left\{\begin{array}{rcl} x_1 & = & 7 - \frac{17}{5}x_3 \\ x_2 & = & 3 - \frac{6}{5}x_3 \\ x_4 & = & 1 \\ & & x_3 \text{ free} \end{array}\right.$$

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