**1S11 (Timoney) Tutorial sheet 6**
[October 30 – November 2, 2012]

**Name:** Solutions

1. Write an augmented matrix corresponding to the following system of linear equations:

$$\begin{array}{rcrcrcrcr} 5x_1 & - & 2x_2 & + & x_3 & - & 4x_4 & = & -3 \\ 2x_1 & + & 3x_2 & + & 7x_3 & + & 2x_4 & = & 18 \\ -x_1 & - & 12x_2 & - & 11x_3 & - & 16x_4 & = & -37 \\ x_1 & + & 2x_2 & - & x_3 & - & x_4 & = & -3 \end{array}$$

*Solution:*

$$\left[\begin{array}{cccccr} 5 & -2 & 1 & -4 & : & -3 \\ 2 & 3 & 7 & 2 & : & 18 \\ -1 & -12 & -11 & -16 & : & -37 \\ 1 & 2 & -1 & -1 & : & -3 \end{array}\right]$$

2. Write out a system of linear equations (in the unknowns $x_1$, $x_2$ and $x_3$) corresponding to the augmented matrix:

$$\left[\begin{array}{rrrcr} 2 & 3 & -2 & : & -12 \\ 3 & 1 & -1 & : & 11 \\ -3 & 2 & 6 & : & -9 \end{array}\right]$$

*Solution:*

$$\begin{array}{rcrcrcr} 2x_1 & + & 3x_2 & - & 2x_3 & = & -12 \\ 3x_1 & + & x_2 & - & x_3 & = & 11 \\ -3x_1 & + & 2x_2 & + & 6x_3 & = & -9 \end{array}$$

3. Use Gaussian elimination (**keeping precisely to the recipe**) followed by back-substitution to solve the system of equations corresponding to the augmented matrix of the previous question.

*Solution:*

$$\left[\begin{array}{rrrcr} 2 & 3 & -2 & : & -12 \\ 3 & 1 & -1 & : & 11 \\ -3 & 2 & 6 & : & -9 \end{array}\right]$$

$$\left[\begin{array}{rrrcr|l} 1 & \frac{3}{2} & -1 & : & -6 & \text{OldRow1} \times \frac{1}{2} \\ 3 & 1 & -1 & : & 11 & \\ -3 & 2 & 6 & : & -9 & \end{array}\right.$$

$$\left[\begin{array}{rrrcr|l} 1 & \frac{3}{2} & -1 & : & -6 & \\ 0 & -\frac{7}{2} & 2 & : & 29 & \text{OldRow2} - 3\text{OldRow1} \\ 0 & \frac{13}{2} & 3 & : & -27 & \text{OldRow3} + 3\text{OldRow1} \end{array}\right.$$

$$\left[\begin{array}{cccc|c} 1 & \frac{3}{2} & -1 & : & -6 \\ 0 & 1 & -\frac{4}{7} & : & -\frac{58}{7} \\ 0 & \frac{13}{2} & 3 & : & -27 \end{array}\right] \text{OldRow2} \times \left(-\tfrac{2}{7}\right)$$

$$\left[\begin{array}{cccc|c} 1 & \frac{3}{2} & -1 & : & -6 \\ 0 & 1 & -\frac{4}{7} & : & -\frac{58}{7} \\ 0 & 0 & \frac{47}{7} & : & \frac{188}{7} \end{array}\right] \text{OldRow3} - \tfrac{13}{2} \times \text{OldRow2}$$

$$\left[\begin{array}{cccc|c} 1 & \frac{3}{2} & -1 & : & -6 \\ 0 & 1 & -\frac{4}{7} & : & -\frac{58}{7} \\ 0 & 0 & 1 & : & 4 \end{array}\right] \text{OldRow3} \times \tfrac{7}{47}$$

Translate to equations now, and do back substitution.

$$\left\{ \begin{array}{rcrcrcr} x_1 & + & \frac{3}{2}x_2 & - & x_3 & = & -6 \\ & & x_2 & - & \frac{4}{7}x_3 & = & -\frac{58}{7} \\ & & & & x_3 & = & 4 \end{array} \right.$$

$$\left\{ \begin{array}{rcl} x_3 & = & 4 \\ x_2 & = & -\dfrac{58}{7} + \dfrac{4}{7}x_3 \\ & = & -\dfrac{58}{7} + \dfrac{16}{7} = \dfrac{42}{7} = -6 \\ x_1 & = & -6 - \dfrac{3}{2}x_2 + x_3 \\ & = & -6 + 9 + 4 = 7 \end{array} \right.$$

Thus we end up with the one solution $x_1 = 7$, $x_2 = -6$ and $x_3 = 4$.

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