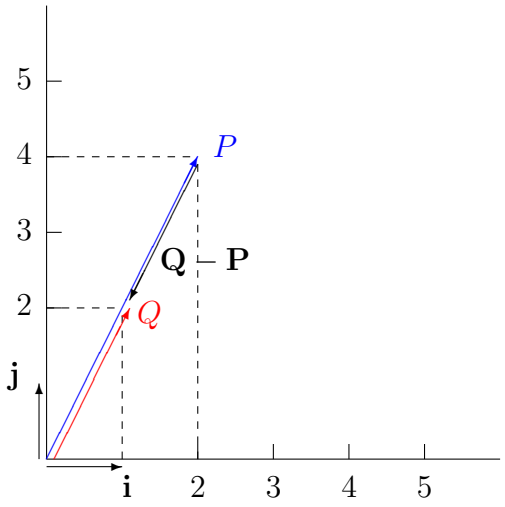
MA1S11 (Timoney) Tutorial/Exercise sheet 1

[due Monday October 1, 2012]

Solutions

1.



(a) Show (on the graph) the point P with coordinates $(2, 4)$ and the point Q with coordinates $(1, 2)$ [and label the points!]

(b) Sketch the position vectors of the two points ($\mathbf{P}$ for P and $\mathbf{Q}$ for Q) [and label them!]

(c) Draw the vector $\mathbf{Q} - \mathbf{P}$

(d) Calculate the distance from P to Q.

Solution:

$$\text{ditsance } (P, Q) = \sqrt{(2-1)^2 + (4-2)^2} = \sqrt{1+4} = \sqrt{5}$$

(e) Calculate $\|\mathbf{Q} - \mathbf{P}\|$.

Solution: Since the vector is represented by the arrow $\vec{PQ}$ its length is the same as the ditance from Q to P, which we have just calculated as $\sqrt{5}$.

2. For $\mathbf{v} = -3\mathbf{i} + 7\mathbf{j}$ and $\mathbf{w} = 6\mathbf{i} - 3\mathbf{j}$, calculate

(a) $\|\mathbf{v} + \mathbf{w}\|$

Solution:

$$\|\mathbf{v} + \mathbf{w}\| = \|(-3+6)\mathbf{i} + (7-3)\mathbf{j}\| = \|3\mathbf{i} + 4\mathbf{j}\| = \sqrt{3^2 + 4^2} = 5$$

(b) The coordinates of the points in the plane with position vectors $\mathbf{v}$ and $\mathbf{w}$. (Write down which is which!)

Solution: $\mathbf{v}$ is the position vector of $(-3, 7)$ and $\mathbf{w}$ is the position vector of $(6, -3)$ and

(c) $\mathbf{v}.\mathbf{w}$

Solution:

$$\mathbf{v}.\mathbf{w} = v_1 w_1 + v_2 w_2 = (-3)(6) + 7(-3) = -18 - 21 = -39$$

(d) $\cos\theta$ where θ is the angle between $\mathbf{v}$ and $\mathbf{w}$.

Solution: We use the formula $\mathbf{v}.\mathbf{w} = \|\mathbf{v}\|\|\mathbf{w}\| \cos\theta$.

We already know $\mathbf{v}.\mathbf{w}$ and we need

$$\begin{aligned} \|\mathbf{v}\| &= \sqrt{(-3)^3 + 7^2} = \sqrt{58} \\ \|\mathbf{w}\| &= \sqrt{6^3 + (-3)^2} = \sqrt{45} \end{aligned}$$

and so we have

$$-39 = \sqrt{58}\sqrt{45}\cos\theta$$

or

$$\cos\theta = -\frac{39}{\sqrt{58}\sqrt{45}} = -\frac{39}{\sqrt{58}(3)\sqrt{5}} = -\frac{13}{\sqrt{58}\sqrt{5}}$$