1S11 (Timoney) Tutorial sheet 11
[December 11 – 14, 2012]

Name: Solutions

1. Convert $(1010011001)_2$ to octal and $(3146)_8$ to binary using the “3 binary for 1 octal digit” rule.

   Solution:

$$(1010011001)_2 = (1\ 010\ 011\ 001)_2 = (001\ 010\ 011\ 001)_2 = (1231)_8$$

$$(3146)_8 = (011\ 001\ 100\ 110)_2 = (011001100110)_2 = (11001100110)_2$$

2. Convert the (base ten) number 923 to octal (by repeated division by 8).

   Solution:

$$\begin{array}{r|r c l}
 & & & \text{Remainder} \\
8 & 923 & & \\ \cline{2-2}
 & 115 & & 3 \\
8 & 115 & & \\ \cline{2-2}
 & 14 & & 3 \\
8 & 14 & & \\ \cline{2-2}
 & 1 & & 6 \\
8 & 1 & & \\ \cline{2-2}
 & 0 & & 1
\end{array}$$

   So 923 is $(1633)_8$.

   Then convert it to binary (using the “3 binary for 1 octal digit” rule)

   Solution:

$$(1633)_8 = (001\ 110\ 011\ 011)_2 = (1110011011)_2$$

   and then from that to hexadecimal.

   Solution:

$$(1633)_8 = (0011\ 10011011)_2 = (39b)_{16}$$

3. Find the mantissa and exponent (**both in binary**) for the binary floating point number $(101011.0111)_2$ when it is converted to (binary) scientific notation.

   Solution:

$$(101011.0111)_2 = (1.010110111)_2 \times 2^5$$

   The mantissa is 1.010110111 and the exponent is 5, which is 101 in binary.

4. With the aid of the following table, show how the integers 13 and -14 would be converted to a bit pattern (zeros and ones) in a computer with 32 bit integers.

13			...						
-14			...						
Bit position:	1	2	...	27	28	29	30	31	32

Solution:

13	0	0	...	0	0	1	1	0	1
-14	1	1	...	1	1	0	0	1	0
Bit position:	1	2	...	27	28	29	30	31	32

5. Convert $\frac{23}{5}$ to binary.

Solution: First $\frac{23}{5} = 4 + \frac{3}{5}$ and $4 = (100)_2$. We concentrate on the fractional part $\frac{3}{5}$.

Imagine the binary expansion as

$$\begin{aligned}
\frac{3}{5} &= (0.b_1b_2b_3\ldots)_2 \\
\text{Double} & \\
\frac{6}{5} &= (b_1.b_2b_3b_4\ldots)_2 \\
\text{Integer parts} & \\
& \boxed{b_1 = 1} \\
\text{Fractional parts} & \\
\frac{1}{5} &= (0.b_2b_3b_4\ldots)_2 \\
\text{Double} & \\
\frac{2}{5} &= (b_2.b_3b_4b_5\ldots)_2 \\
\text{Integer parts} & \\
& \boxed{b_2 = 0} \\
\text{Fractional parts} & \\
\frac{2}{5} &= (0.b_3b_4b_5\ldots)_2 \\
\text{Double} & \\
\frac{4}{5} &= (b_3.b_4b_5b_6\ldots)_2 \\
\text{Integer parts} & \\
& \boxed{b_3 = 0} \\
\text{Fractional parts} & \\
\frac{4}{5} &= (0.b_4b_5b_6\ldots)_2 \\
&= (0.b_1b_2b_3\ldots)_2 \\
\text{Double} & \\
\frac{8}{5} &= (b_4.b_5b_6b_7\ldots)_2 \\
\text{Integer parts} & \\
& \boxed{b_4 = 1}
\end{aligned}$$

$$\begin{aligned}
\text{Fractional parts} & \\
\frac{3}{5} &= (0.b_5b_6b_7\ldots)_2 \\
&= (0.b_1b_2b_3\ldots)_2
\end{aligned}$$

Thus the pattern repeats, $b_5 = b_1$, $b_6 = b_2$, etc and so $\frac{3}{5} = (0.\overline{1001})_2$. The answer is

$$\frac{23}{5} = 4 + \frac{2}{7} = (100.\overline{1001})_2$$

6. Suppose a computer was built that used 48 bits to store each floating point number, with 1 bit for the sign, 16 bits for the exponent (giving a range of exponents from -2^{15} to $2^{15} - 1$) and the remaining 31 bits for the mantissa. Roughly (using standard base ten exponential notation) what would be the biggest positive number the computer could store using this system? [Hint: $2^{16} = 65536$.]

   Solution: The largest positive number would be $(1.111\ldots1)_2 \times 2^{32768-1}$ with 31 ones in the mantissa (and largest possible exponent $2^{15} - 1 = 32768 - 1 = 32767$. So just about

$$\begin{aligned} 2 \times 2^{32768-1} &= 2^{32768} = 2^8 \times 2^{32760} = 2^8 \times (2^{10})^{3276} \\ &\cong 256 \times (10^3)^{3276} = 2.56 \times 10^2 \times 10^{9828} = 2.56 \times 10^{9830} \end{aligned}$$

   And what would be the smallest positive number?

   Solution: The smallest would be 1×2^{-32768} (smallest mantissa 1 and most negative exponeint $-2^{15} = -32768$).

$$2^{-32768} = 2^2 \times 2^{-32770} = 2^2 \times (2^{10})^{-3277} \cong 2^2 \times (10^3)^{-3277} = 4^2 \times 10^{-9831}$$

Richard M. Timoney