

**Coláiste na Tríonóide, Baile Átha Cliath Trinity College Dublin** Ollscoil Átha Cliath | The University of Dublin

# Faculty of Engineering, Mathematics and Science

# School of Mathematics

JF Engineering JF Engineering with Management JF MSISS

Trinity Term 2016

MA1E02 — Engineering Mathematics II

Thursday, May 12 RDS 14.00 — 16.00

Professor R. M. Timoney

## Instructions to Candidates:

Please attempt all questions.

All questions have equal weight (10 points each).

'Formulae & tables' are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used.

You may not start this examination until you are instructed to do so by the Invigilator.

## With Answers

1. Find the equation of the plane in space that contains the 3 points (1, 2, 3), (2, 0, 1) and (0, 3, 4).

Anwswer (without workings):

$$y - z = -1.$$

(The idea is that if we call the points A, B and C, then the vectors  $\vec{AB} = \mathbf{B} - \mathbf{A}$  and  $\vec{AC} = \mathbf{C} - \mathbf{A}$  lie in the plane and their cross product  $\vec{AB} \times \vec{AC}$  gives a normal vector to the plane. That should be  $\mathbf{j} - \mathbf{k}$  in this case. So the equation is 0x + y - z = c and you should get c = 1 from any of the points A, B or C.)

2. Find parametric equations for the line with vector equation

$$x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = 4\mathbf{i} - 2\mathbf{k} + t(-\mathbf{i} + 4\mathbf{j})$$

Anwswer (without workings):

$$\begin{cases} x = 4 - t \\ y = 4t \\ z = -2 \end{cases}$$

(Compare the coefficients of i, j and k on the left and right.)

3. Use an appropriate sustitution to evaluate

$$\int 3x e^{-17x^2} \, dx$$

Anwswer (without workings):

$$-\frac{3}{34}e^{-17x^2} + C$$

(You can put  $u = -17x^2$ , du = -34x dx.)

4. Calculate

$$\int_0^{\pi/3} \cos^2(2x) \, dx$$

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Anwswer (without workings):

$$\frac{\pi}{6} - \frac{\sqrt{3}}{16}$$

(Use  $\cos^2(2x) = \frac{1}{2}(1 + \cos(4x))$ .)

5. Use partial fractions to evaluate

$$\int \frac{2x^2 - 9x - 9}{x^3 - 9x} \, dx$$

Anwswer (without workings):

$$\ln|x| - \ln|x - 3| + 2\ln|x + 3| + C$$

(Partial fractions form is

$$\frac{2x^2 - 9x - 9}{x^3 - 9x} = \frac{2x^2 - 9x - 9}{x(x - 3)(x + 3)} = \frac{A_1}{x} + \frac{A_2}{x - 3} + \frac{A_3}{x + 3} \qquad \Big)$$

6. Find the solution of the (first order linear) differential equation

$$\frac{dy}{dx} - 4y = e^{3x}$$

with y = 2 at x = 0.

Anwswer (without workings):

$$y = -e^{3x} + 3e^{4x}$$

(Integrating factor  $e^{-4x}$ .)

7. Use the integral test to determine whether the series

$$\sum_{n=1}^{\infty} \frac{1}{(n+1)\ln(n+1)}$$

converges or not.

Anwswer (without workings):

$$\int_{1}^{\infty} \frac{1}{(x+1)\ln(x+1)} \, dx = \infty$$

and so it does not converge.

(For the integral  $u = \ln(x+1)$ ,  $du = \frac{1}{x+1} dx$  works and an antiderivative is  $\ln(\ln(x+1))$ .) Page 3 of 5

8. Use sigma notation to write the Taylor series for  $e^x$  about  $x_0 = 2$ .

Anwswer (without workings):

$$\sum_{n=0}^{\infty} \frac{e^2}{n!} (x-2)^n$$

9. For the following system of linear equations:

$$3x_2 + 12x_3 + 6x_4 = -5$$
  
- 3x<sub>2</sub> + 24x<sub>3</sub> - 2x<sub>4</sub> = -53  
- 6x<sub>2</sub> - 36x<sub>3</sub> - 4x<sub>4</sub> = 62  
2x<sub>1</sub> - 9x<sub>2</sub> - 12x<sub>3</sub> - 6x<sub>4</sub> = 9

(a) [5 points] Write an augmented matrix for the system of equations.

Anwswer (without workings):

(b) [5 points] Following the method of Gauss-Jordan elimination strictly, reduce the augmented matrix to reduced row echelon form.

Anwswer (without workings): (Method is important, not just the answer.)

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1	0	0	0	:	(	)	
0	1	0	0	:	-2	2/3	
0	0	1	0	:	_	-2	
0	0	0	1	:	7,	7/2	
			1	0	2		
A =		8	0	9			
			1	11	9		

10. Let

Find the inverse 
$$A^{-1}$$
 (if it exists).

Anwswer (without workings):

$$A^{-1} = \begin{bmatrix} -\frac{9}{7} & \frac{2}{7} & 0\\ -\frac{9}{11} & \frac{1}{11} & \frac{1}{11}\\ \frac{8}{7} & -\frac{1}{7} & 0 \end{bmatrix}$$