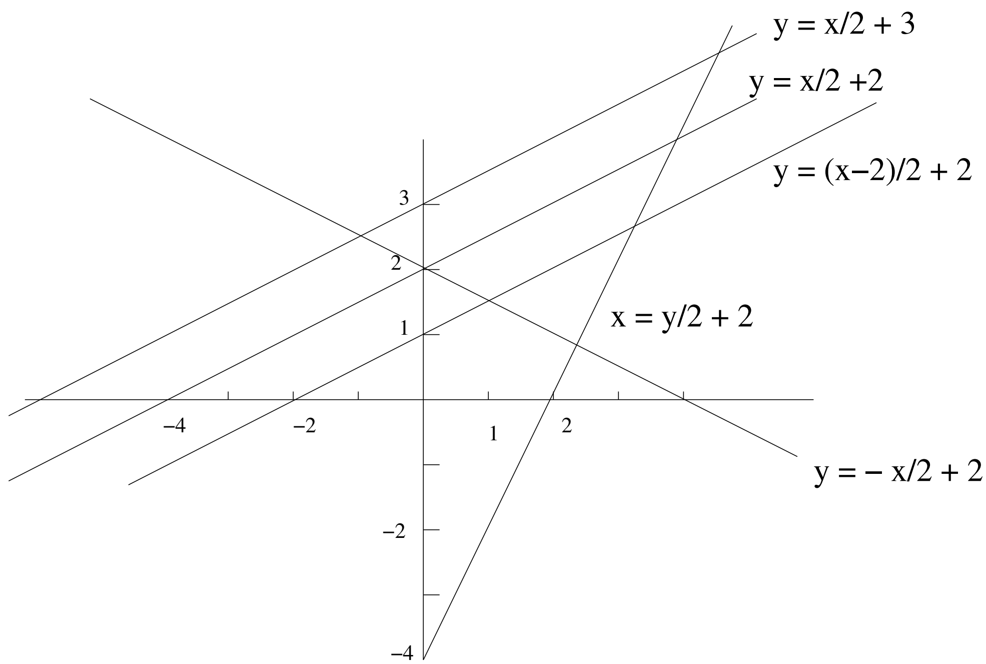
MA1311 (Advanced Calculus) Tutorial sheet 1

[September 30 – October 1, 2010]

Name: Solutions

1. Plot the graph of the (line) $y = \frac{1}{2}x + 2$

   Solution:



   Then plot (on the same picture, but label the different plots clearly) $y = \frac{1}{2}(x-2)+2$, $y = -\frac{1}{2}x + 2$, $y = 1 + \frac{1}{2}x + 2$ and $x = \frac{1}{2}y + 2$.

2. For a graph $y = f(x)$ (of a function with a domain contained in $\mathbb{R}$ and with real values) how is the graph $y = f(-x)$ related to that of $y = f(x)$? [Hint: Look at the previous question.]

   Solution: The graph of $y = f(-x)$ is the reflection in the y-axis of the graph $y = f(x)$.

   How is the graph of $y = f(x) + 7$ related to the graph of $y = f(x)$?

   Solution: The graph of $y = f(x) + 7$ can be got by moving the graph $y = f(x)$ up by 7 units (or by moving the x-axis down 7 units).

   How is the graph of $y = f(x + 7)$ related to the graph of $y = f(x)$?

   Solution: The graph of $y = f(x + 7)$ is the graph $y = f(x)$ moved by 7 units to the left (or -7 units to the right).

How is the graph of $y = 6f(x)$ related to the graph of $y = f(x)$?

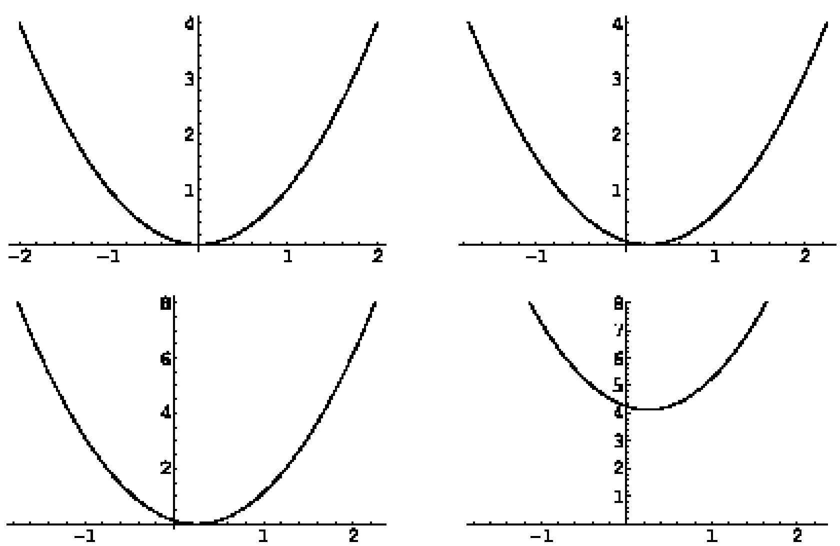
Solution: The graph of $y = 6f(x)$ can be got from the graph $y = f(x)$ by multiplying the y-coordinates of all the points by 6 (or by changing the scale on the y-axis so that 1 in the old scale becomes 6 in the new scale).

3. Starting from the graph of $y = x^2$, find the graph of $y = 2x^2 - x + 4$. [Hint: Complete the square on the x^2 and x terms.]

   Solution: We can write

$$\begin{aligned} y &= 2x^2 - x + 4 \\ &= 2\left(x^2 - \frac{1}{2}x\right) + 4 \\ &= 2\left(x^2 - \frac{1}{2}x + \left(\frac{1}{4}\right) + \frac{1}{16}\right) + 4 \\ &= 2\left(x - \frac{1}{4}\right)^2 + \frac{33}{8} \end{aligned}$$

   The graph $y = x^2$ is a parabola opening up from the origin. To get the graph $y = (x-1/4)^2$ we should move that parabola $1/4$ to the right.



   Then magnify the y-coordinates by a factor 2 (or change the y-scale so that 1 becomes labelled 2), and finally shift up by $33/8$.

4. Consider functions f and g given by the formulae:

$$f(x) = \frac{1}{\sqrt{x-2}}, \qquad g(x) = \frac{1}{\sqrt{x^2-1}}.$$

Express the 'natural' domain for f and g using interval notation (and set theoretic symbols like $\cup$ for union).

Solution: In order for $\sqrt{x-2}$ to make sense (as a real number) we need $x - 2 \geq 0$ or $x \geq 2$. But to avoid division by zero we need to exclude $x = 2$. So we get $x > 2$.

domain of f is: $(2, \infty)$

For $\sqrt{x^2 - 1}$ we need $x^2 - 1 \geq 0$, which means $x^2 \geq 0$ or $|x| \geq 1$. This happens for $x \leq -1$ and for $x \geq 1$. So we need $x \in (-\infty, -1] \cup [1, \infty)$. But we also have to exclude division by zero, which means excluding $x = \pm 1$.

domain of g is: $(-\infty, -1) \cup (1, \infty)$.

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