MA1311 (Advanced Calculus) Exercise sheet 12

[Due Monday January 17th, 2011]

Name:

Student ID:

1. Express the volume of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1$$

as an iterated integral (*i.e.* set up the triple integral in terms of single integrals, but do not evaluate it).

2. Evaluate

$$\int_0^3 \int_{y=0}^{y=\sqrt{3}x} \frac{1}{\sqrt{x^2+y^2}}\, dy\, dx$$

by making a change of variables to polar coordinates. [Hint: Sketch the region first. Then do the dr integral first, before $d\theta$. $\int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + C$]

3. Find the mass of a solid object occupying the region in space bounded by the coordinate planes and the plane $x + y + z = 2$ if its density function is $\delta(x, y, z) = x^2$. [Hint: The mass is the triple integral of the density over the region. Calculations are easier if you leave the dx integral to last.]

Please hand in your work at the School of Mathematics office. Richard M. Timoney