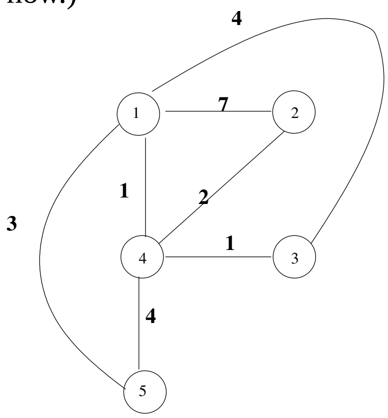
3E1 Trinity Term Tutorial sheet 8

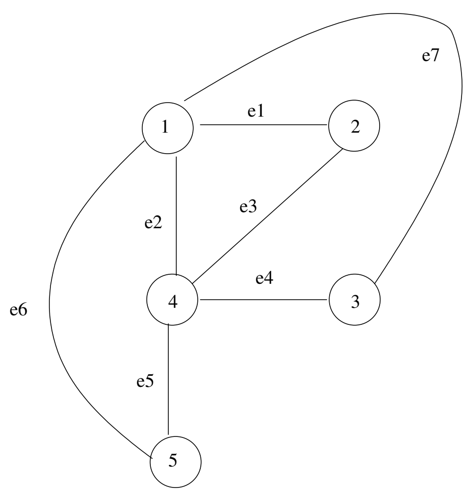
[April 2 – 7, 2003]

Name: Solutions

1. For the following graph, write out the incidence table. (Ignore the edge "lengths" given for now.)



Solution:



vertex / edge	1	2	3	4	5
1	1	1	0	0	0
2	1	0	0	1	0
3	0	1	0	1	0
4	0	0	1	1	0
5	0	0	0	1	1
6	1	0	0	0	1
7	1	0	1	0	0

2. Use Dijkstra's algorithm to find the shortest lengths from vertex 1 to each of the other vertices j for the graph in the above picture.

Solution:

Step 0 (preliminary): Vertex 1 gets permanent shortest length $L_1 = 0$. Other vertices get temporary lengths $TL_2 = 7$, $TL_3 = 4$, $TL_4 = 1$ and $TL_5 = 3$.

Define the permanent lengths set as $\mathcal{PL} = \{1\}$ and the temporary lengths set as $\mathcal{TL} = \{2, 3, 4, 5\}$.

Step 1: $j = 4$ has the smallest TL_j, $j \in \mathcal{TL}$. $L_4 = 1$, $\mathcal{PL} = \{1, 4\}$ and $\mathcal{TL} = \{2, 3, 5\}$.

As $\mathcal{TL}$ is not the empty set, continue to step 2.

Step 2: For each j in $\mathcal{TL}$, update TL_j:

$$\begin{aligned} TL_2 &= \min(TL_2, L_1 + \ell_{12}, L_4 + \ell_{42}) \\ &= \min(7, 0 + 7, 1 + 2) \\ &= 3 \\ TL_3 &= \min(TL_3, L_1 + \ell_{13}, L_4 + \ell_{43}) \\ &= \min(4, 0 + 4, 1 + 1) \\ &= 2 \\ TL_5 &= \min(TL_5, L_1 + \ell_{15}, L_4 + \ell_{45}) \\ &= \min(3, 0 + 3, 1 + 4) \\ &= 3 \end{aligned}$$

Step 1: $j = 3$ has the smallest TL_j, $j \in \mathcal{TL}$. $L_3 = 2$, $\mathcal{PL} = \{1, 3, 4\}$ and $\mathcal{TL} = \{2, 5\}$.

As $\mathcal{TL}$ is not the empty set, continue to step 2.

Step 2: For each j in $\mathcal{TL}$, update TL_j:

$$\begin{aligned} TL_2 &= \min(TL_2, L_1 + \ell_{12}, L_3 + \ell_{32}, L_4 + \ell_{42}) \\ &= \min(3, 0 + 3, 2 + \infty, 1 + 2) \\ &= 3 \\ TL_5 &= \min(TL_5, L_1 + \ell_{15}, L_3 + \ell_{35}, L_4 + \ell_{45}) \\ &= \min(3, 0 + 3, 2 + \infty, 1 + 4) \\ &= 3 \end{aligned}$$

Step 1: $j = 2$ has the smallest TL_j, $j \in \mathcal{TL}$. (So has $j = 5$ but we choose the smaller j according to the algorithm.) $L_2 = 3$, $\mathcal{PL} = \{1, 2, 3, 4\}$ and $\mathcal{TL} = \{5\}$.

As $\mathcal{TL}$ is not the empty set, continue to step 2.

Step 2: For each j in $\mathcal{TL}$, update TL_j:

$$\begin{aligned} TL_5 &= \min(TL_5, L_1 + \ell_{15}, L_2 + \ell_{25}, L_3 + \ell_{35}, L_4 + \ell_{45}) \\ &= \min(3, 0 + 3, 3 + \infty, 2 + \infty, 1 + 4) \\ &= 3 \end{aligned}$$

Step 1: $j = 5$ has the smallest TL_j, $j \in \mathcal{TL}$. $L_5 = 3$, $\mathcal{PL} = \{1, 2, 3, 4, 5\}$ and $\mathcal{TL} = \emptyset$.

As $\mathcal{TL}$ is now empty, we finish with output

$$L_1 = 0, L_2 = 3, L_3 = 2, L_4 = 1, L_5 = 3.$$