3E1 Hilary/Trinity Term Tutorial sheet 7
[March 5 – 31, 2003]

Name: Solutions

1. Use the simplex algorithm to maximise $z = 900x_1 + 800x_2 + 300x_3$ subject to

$$\begin{array}{rcrcrcrcrcl} 3x_1 & + & 2x_2 & + & x_3 & + & x_4 & & & = & 60 \\ x_1 & + & x_2 & & & & & + & x_5 & = & 15 \end{array}$$

and $x_j \geq 0$ for $1 \leq j \leq 5$. [Note: This is the continuation of the problem at the end of the previous tutorial sheet.]

Solution: Writing things as a tableau, we get

$$\left[\begin{array}{c|rrrrr|r} 1 & -900 & -800 & -300 & 0 & 0 & 0 \\ \hline 0 & 3 & 2 & 1 & 1 & 0 & 60 \\ 0 & 1 & 1 & 0 & 0 & 1 & 15 \end{array}\right]$$

Choose the pivot column as 2 (the one with -900 at the top, most negative) and then for the row of the pivot entry we look at the ratios $60/3 = 20$, $15/1 = 15$ of the right hand sides to the (positive) entries in the column. We choose the least of these anad pivot on the $(3, 2)$ entry. This is already 1 [no need to divide across the row by it] and so we do Row 1 $+900$ Row 3, Row 2 -3 Row 3, to get

$$\left[\begin{array}{c|rrrrr|r} 1 & 0 & 100 & -300 & 0 & 900 & 13500 \\ \hline 0 & 0 & -1 & 1 & 1 & -3 & 15 \\ 0 & 1 & 1 & 0 & 0 & 1 & 15 \end{array}\right]$$

Next pivot column is Column 4 (-300) and Row 2 (only $15/1$ to consider). Pivot on entry $(2, 4)$. We do Row 1 $+300$ Row 2 and get

$$\left[\begin{array}{c|rrrrr|r} 1 & 0 & -200 & 0 & 300 & 0 & 18000 \\ \hline 0 & 0 & -1 & 1 & 1 & -3 & 15 \\ 0 & 1 & 1 & 0 & 0 & 1 & 15 \end{array}\right]$$

Next pivot column is 3 (-200) and Row 3. Pivot on $(3, 3)$ entry. Row 1 $+200$ Row 3, Row 2 $+$ Row 3.

$$\left[\begin{array}{c|rrrrr|r} 1 & 200 & 0 & 0 & 300 & 200 & 21000 \\ \hline 0 & 1 & 0 & 1 & 1 & -2 & 30 \\ 0 & 1 & 1 & 0 & 0 & 1 & 15 \end{array}\right]$$

The basic variables are now x_2 and x_3 (where there are zeros in the objective function row (Row 1) and a permuted identity matrix in the remaining columns excluding the right hand sides column). The objective function has value $z = 21000 - (200x_1 + 300x_4 + 200x_5)$ which is never more than 21000 and is 21000 at $x_1 = x_4 = x_5 = 0$ [non-basic equal to zero]. Corresponding values of x_2 and x_3 are $x_3 = 30$, $x_2 = 15$.

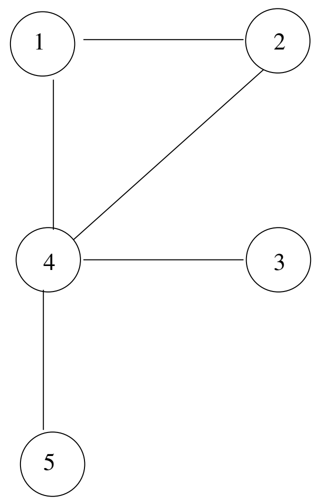
So the maximum is 21000 at $(x_1, x_2, x_3, x_4, x_5) = (0, 15, 30, 0, 0)$.

[In the original problem, x_4 and x_5 were the slack variables.]

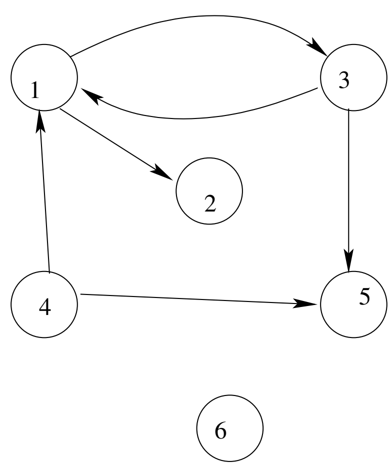
2. The following is the adjacency matrix for a graph. Sketch the graph.

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Solution:



3. For the following directed graph, write out the adjacency matrix.



Solution: Edge *from* vertex i *to* vertex j gives 1 in (i, j) entry.

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$