3E1 Hilary Term Tutorial sheet 6
[February 26 – March 3, 2003]

Name:

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Course: (Circle one) Eng MEMS MSISS

1. Is the following PDE elliptic, hyperbolic or parabolic?

$$\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + 3\frac{\partial^2 u}{\partial y^2} + 4\frac{\partial u}{\partial x} + 5\frac{\partial u}{\partial y} = x^2$$

Solution: Taking $A = 1$, $B = 2/2 = 1$, and $C = 3$ we have the second order terms in the form

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + B\frac{\partial^2 u}{\partial y \partial x} + C\frac{\partial^2 u}{\partial y^2} = A\frac{\partial^2 u}{\partial x^2} + 2B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2}$$

The criterion for classifying the PDE depends on the sign of

$$\det \begin{bmatrix} A & B \\ B & C \end{bmatrix} = AC - B^2 = 3 - 1 = 2 > 0$$

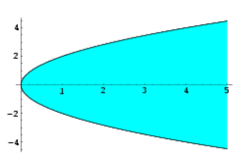
So the equation is elliptic.

2. Find the regions of $(x, t) \in \mathbb{R}^2$ where the following PDE for $u(x, t)$ is elliptic

$$\frac{\partial^2 u}{\partial x^2} + t\frac{\partial^2 u}{\partial x \partial t} + x\frac{\partial^2 u}{\partial t^2} = 0$$

Solution: As in the previous problem $A = 1$, $B = t/2$ and $C = x$. So the equation is elliptic in that part of $\mathbb{R}^2$ where $AC - B^2 = x - t^2/4 > 0$, or $x > t^2/4$.

(Graphically, $x = t^2/4$ is a parabola symmetric around the x-axis and our region is to the right of [within?] the parabola if we take the x-axis as horizontal.)



3. A publisher has 3 titles available for printing today, B1, B2 and B3. Total paper supplies are limited to 60 units and the 3 books require different amounts of paper. They also give different profits as follows.

	B1	B2	B3
Units of paper per 1000 copies	3	2	1
Profit per 1000 copies (€)	900	800	300

Because they attract similar readers, it is estimated that the combined sales of B1 and B2 will not exceed 15000 copies.

Set up the problem of maximising the profit for these items via an objective function to be maximised subject to inequalities. Then convert this problem to a standard form for the simplex algorithm (introduce slack variables to convert the constraints to a system of equations to be satisfied by nonnegative variables). *You are not asked to do the simplex algorithm.*

Solution: It is probably more convenient to work with thousands of books. Let us say that we will have x (thousands) of B1, y of B2, z of $B3$. The constraint on paper says we must have $3x + 2y + z \leq 60$ and the sales/marketing restriction says $x + y \leq 15$ (working in thousands).

The problem is then to maximize the profit $p = 900x + 800y + 300z$ (our *objective function* in this problem) subject to

$$\begin{array}{rcrcrcr} 3x & + & 2y & + & z & \leq & 60 \\ x & + & y & & & \leq & 15 \end{array}$$

plus the restrictions $x \geq 0$, $y \geq 0$ and $z \geq 0$ (which arise because we cannot make a negative number of books).

If we introduce slack variables for the 2 constraints, say $r = 60 - (3x + 2y + z)$ and $s = 15 - (x + y)$, we get

$$\begin{array}{rcrcrcrcrcr} 3x & + & 2y & + & z & + & r & & & = & 60 \\ x & + & y & & & & & + & s & = & 15 \end{array}$$

and the problem is to maximise $p = 900x + 800y + 300z$ subject to x, y, z, r, s satisfying these 2 equations and being all ≥ 0.

Remark We might be more wise to use subscripts for the variables and slack variables, rather than using a somewhat random choice of letters. So we might use x_1, x_2 and x_3 for the numbers of B1, B2, B3 (in thousands) and then call the slack variables x_4, x_5. we would then get this problem (same but in different notation).

Maximise $900x_1 + 800x_2 + 300x_3$ subject to

$$\begin{array}{rcrcrcrcrcr} 3x_1 & + & 2x_2 & + & x_3 & + & x_4 & & & = & 60 \\ x_1 & + & x_2 & & & & & + & x_5 & = & 15 \end{array}$$

and $x_j \geq 0$ for $1 \leq j \leq 5$.