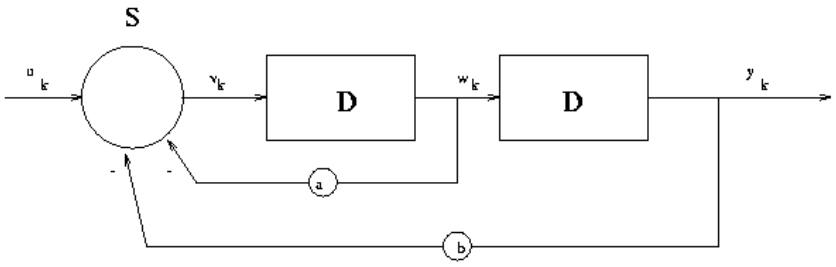
2E2 Tutorial sheet 8 Solution

[Wednesday January 10th, 2001]

1. Draw a block diagram represeneting the discrete time system

$$y_{k+2} + 0.5y_{k+1} + 0.25y_k = u_k.$$

Solution:



This diagram has

$$\begin{aligned} v_k &= u_k - aw_k - by_k \\ v_k &= w_{k+1} \\ w_k &= y_{k+1} \end{aligned}$$

Consequently, we have

$$\begin{aligned} v_k &= y_{k+2} \\ y_{k+2} &= u_k - ay_{k+1} - by_k \\ y_{k+2} + ay_{k+1} + by_k &= u_k \\ u_k &= y_{k+2} + ay_{k+1} + by_k \end{aligned}$$

and this is the right equation if $a = 0.5$ and $b = 0.25$.

2. Which of the following discrete linear systems are stable (v_k represents the input, y_k the output at step k).

(a)

$$2y_{k+2} + 3y_{k+1} - y_k = v_k$$

Solution: The $\mathcal{Z}$ transfer function for this is

$$\frac{1}{2z^2 + 3z - 1}$$

and this has poles (zeroes of the denominator) at

$$z = \frac{-3 \pm \sqrt{9+8}}{4} = \frac{-3 \pm \sqrt{17}}{4}$$

These are $z = -1.78$ and $z = .28$ and so one of them has $|z| = 1.78 > 1$. Hence the system is unstable.

(b)

$$9y_{k+2} + 9y_{k+1} + 2y_k = v_k$$

Solution: The $\mathcal{Z}$ transfer function for this is

$$\frac{1}{9z^2 + 9z + 1}$$

and this has poles (zeroes of the denominator) at

$$z = -frac-9 \pm \sqrt{81 - 72}18 = \frac{-9 \pm \sqrt{9}}{18} = \frac{-9 \pm 3}{18} = \frac{-3 \pm 1}{6}$$

which are $z = -1/3$ and $z = -2/3$. Both of these have $|z| < 1$ and so the system is stable.