Some Hilbertian Operator spaces

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Joint work with L. Bunce continuing earlier work with B. Feely

Operator space

Banach space E together with norms $\|\cdot\|_n$ induced on matrix spaces $M_n(E)$ $(n = 1, 2, \ldots)$ from an isometric embedding $E \overset{\phi}{\hookrightarrow} \mathcal{B}(H)$ and

$$M_n(E) \overset{\phi^{(n)}}{\hookrightarrow} M_n(\mathcal{B}(H)) \equiv \mathcal{B}(H^n) : (x_{i,j})_{i,j=1}^n \mapsto (\phi(x_{i,j}))_{i,j=1}^n$$

For $T: E \to F$ linear, E and F operator spaces, T is completely contractive if

$$\|T^{(n)}(x)\|_n \leq \|x\|_n \qquad (x \in M_n(E), n \geq 1).$$

T is completely isometric if equality holds.
We call E Hilbertian if E is isometric to a Hilbert space.

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Algebra?

An operator space E is called injective if E-valued complete contractions extend.

Theorem. $E = \mathcal{B}(H)$ is an injective operator space.

- For each (concrete) operator space $E \subseteq \mathcal{B}(H)$, there exist minimal injective $I(E)$ with $E \subseteq I(E) \subseteq \mathcal{B}(H)$.
- $E \hookrightarrow I(E)$ is called an injective envelope of E.
- $I(E)$ is a unique minimal containing injective operator space up to commuting diagrams.
- $I(E)$ always has a ternary algebraic structure (TRO, ternary ring of operators).

$T \subset \mathcal{B}(H)$ is called a (concrete) TRO if
$x, y, z \in T \Rightarrow [x, y, z] \stackrel{def}{=} xy^*z \in T$.

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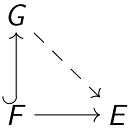

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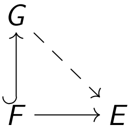

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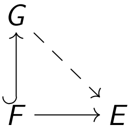

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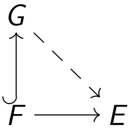

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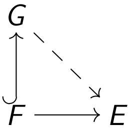

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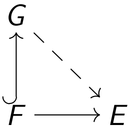

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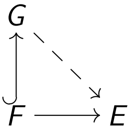

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Theorem. TRO morphisms are complete contractions, hence TRO isomorphisms are complete isometries.

Conversely, surjective complete isometries between TROs are TRO isomorphisms.

Corollary. An abstract TRO (*i.e.* a Banach space T with a ternary operation $[\cdot,\cdot,\cdot]$ that arises from an isometry onto a concrete TRO) has a canonical operator space structure.

Given an (abstract) operator space E, its triple envelope is $\mathcal{T}(E) =$ the TRO generated by E inside $I(E)$. [Hamana]

Universal property: 'smallest' TRO generated by completely isometric embeddings $E \hookrightarrow \mathcal{B}(H)$ (in a sense of quotient TRO morphism)

Like $E \hookrightarrow I(E)$, $E \hookrightarrow \mathcal{T}(E)$ is unique up to diagram chasing.

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Hilbertian operator spaces

Many different examples.

Row Hilbert space

$$M_{1,d}(\mathbb{C}) \subseteq M_d(\mathbb{C}) \cong \mathcal{B}(\ell_2^d)$$

and column Hilbert space

$$M_{d,1}(\mathbb{C}) \subseteq M_d(\mathbb{C}) \cong \mathcal{B}(\ell_2^d)$$

are TROs and injective operator spaces (but not completely isometric for any $2 \leq d \leq \infty$).

These are the only Hilbertian TROs.

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JC^*-triples

A (concrete) JC^*-triple is a closed $E \subseteq \mathcal{B}(H)$ such that

$$a, b, c \in E \Rightarrow \{a, b, c\} \stackrel{def}{=} \frac{1}{2}([a, b, c] + [c, b, a]) \in E$$

Surjective linear isometries between JC^*-triples are isomorphisms of the triple product structure (and conversely).

An abstract JC^*-triple $(E, \{\cdot, \cdot, \cdot\})$ is a Banach space E and a triple product $(a, b, c) \in E \times E \times E \mapsto \{a, b, c\} \in E$ on it that arises from some isometric embedding of E as a concrete JC^*-triple.

Examples.

- Row and column Hilbert space are the 'same' as JC^*-triples (but not as operator spaces).
- $$E = \{x \oplus x^t : x \in M_{1,d}(\mathbb{C})\} \subset M_d \oplus M_d \subset M_{2d}$$

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A (concrete) JC^*-triple is a closed $E \subseteq \mathcal{B}(H)$ such that

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Surjective linear isometries between JC^*-triples are isomorphisms of the triple product structure (and conversely).
An abstract JC^*-triple $(E, \{\cdot, \cdot, \cdot\})$ is a Banach space E and a triple product $(a, b, c) \in E \times E \times E \mapsto \{a, b, c\} \in E$ on it that arises from some isometric embedding of E as a concrete JC^*-triple.
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Theorem. Given an abstract JC^*-triple E, there exists a universal (largest) TRO $T^*(E)$ generated by E.

More precisely, there exists an isometric embedding $E \overset{\alpha_E}{\hookrightarrow} T^*(E)$ into a TRO $T^*(E)$ with the universal property

where $\pi \colon E \to T$ is any given triple morphism
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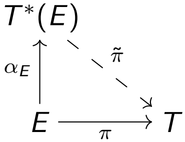

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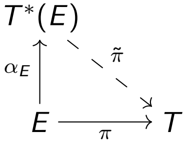

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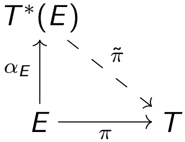

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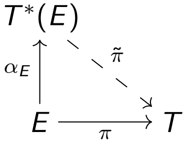

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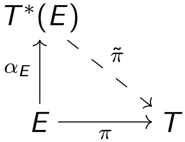

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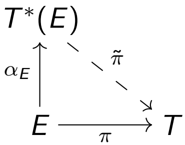

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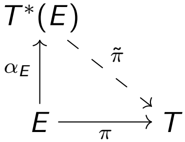

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Operator space structures of a JC^*-triple

Definition. Given a JC^*-triple E, a JC-operator space structure on E is an operator space structure induced by an isometric embedding $\pi\colon E \to \mathcal{B}(H)$ onto a concrete JC^*-triple $\pi(E) \subseteq \mathcal{B}(H)$.

Corollary. For each JC-operator space structure on E, there exists a TRO ideal $\mathcal{I} \subset T^*(E)$ with $\mathcal{I} \cap \alpha_E(E) = \{0\}$ such that E is completely isometric to $E_{\mathcal{I}}$, the operator space structure on E determined by the isometric embedding $E \to T^*(E)/\mathcal{I}$ $(x \mapsto \alpha_E(x) + \mathcal{I})$

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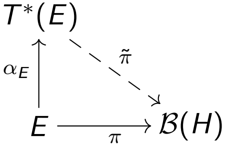

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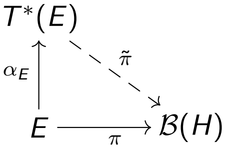

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Hilbertian JC-operator spaces (2)

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Proposition. (follows via Lemma of Le Merdy, Ricard, Roydor) E Hilbertian JC-operator space of dimension $d \Rightarrow E$ is a homogeneous operator space. Thus all hyperplanes are completely isometric.

Proposition If $E = E_{\mathcal{I}(S)}$ for $S \subsetneq \{1, 2, \ldots, d\}$ and F is a hyperplane in E, then $F = F_{\mathcal{I}(\sigma)}$ where $\sigma \subsetneq \{1, 2, \ldots, d-1\}$ is $\sigma = \{r : r, r+1 \in S\}$.

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Let $E \subset \mathcal{B}(H)$ be an infinite-dimensional Hilbertian JC^*-subtriple and operator space.

- Then $T^*(E)$ is the TRO generated by $\{a(x) : x \in E\}$ (inside $\mathcal{B}(\mathscr{F}_-(E))$)
- Either there exists $i, j \geq 0$ with $i + j > 0$ such that d-dimensional subspaces F of E have $F = F_{[i+1,d-j]}$ whenever $d > i + j$ **or** $E = E_{\{0\}}$ (so $\mathrm{TRO}(E) = T^*(E)$).
- If $j = 0$, E is completely isometric to the subspace of $\mathcal{B}(\Lambda^i E, \Lambda^{i-1} E)$ given by the restrictions of annihilation operators.
- Moreover $I(E) = \mathcal{B}(\Lambda^i E, \Lambda^{i-1} E)$ (which is much bigger than $\mathcal{T}(E)$ if $i > 1$).
- For $i = 0$, $I(E)$ is $\mathcal{B}(\Lambda^{j-1} E, \Lambda^j E)$.
- For $\min(i, j) > 0$, $I(E)$ is $\mathcal{B}(\Lambda^i E, \Lambda^{i-1} E) \oplus \mathcal{B}(\Lambda^{j-1} E, \Lambda^j E)$.

Question: identify $I(E_{\{0\}})$?

Envelopes for Hilbertian JC-operator spaces

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Envelopes for Hilbertian JC-operator spaces

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